



Beijing-Dublin International College



SEMESTER II FINAL EXAMINATION – 2022/2023

**Beijing-Dublin International College
BDIC1043J Maths 4 (Advanced Mathematics; Finance)**

PRINCIPAL OF COLLEGE: Wenying WU
MODULE LECTURER: Xin LIU

Time Allowed: 95 minutes

Instructions for Candidates

Your answers to ALL the questions should be written on the Answer Book.

BJUT Student ID: _____ **UCD Student ID:** _____

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge: _____ **(Signature)**

Instructions for Invigilators

No calculators are permitted. NO dictionaries are permitted.
No rough-work paper is to be provided for candidates.

NOTE: The exam paper has **3** sections on **8** pages, with a full score of 100 marks.

SECTION A — MULTIPLE CHOICE QUESTIONS

In each question, choose **at most one** option.

Write your answers on the **Answer Book** provided.

This section is worth a total of **15** marks, with each question worth **3** marks.

1. Evaluate the triple integral

$$I = \int_0^2 dr \int_0^{\frac{\pi}{2}} d\phi \int_0^{\frac{\pi}{2}} d\theta \, r^2 \sin \phi.$$

- (a) 0; (b) $\frac{16\pi}{3}$; (c) $\frac{8\pi}{3}$; (d) $\frac{4\pi}{3}$.

2. For an original photo



the low pass filtered result of Fourier transform is _____ below.



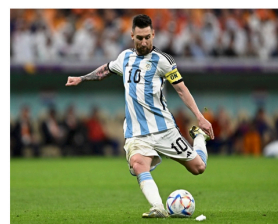
(a)



(b)



(c)



(d)

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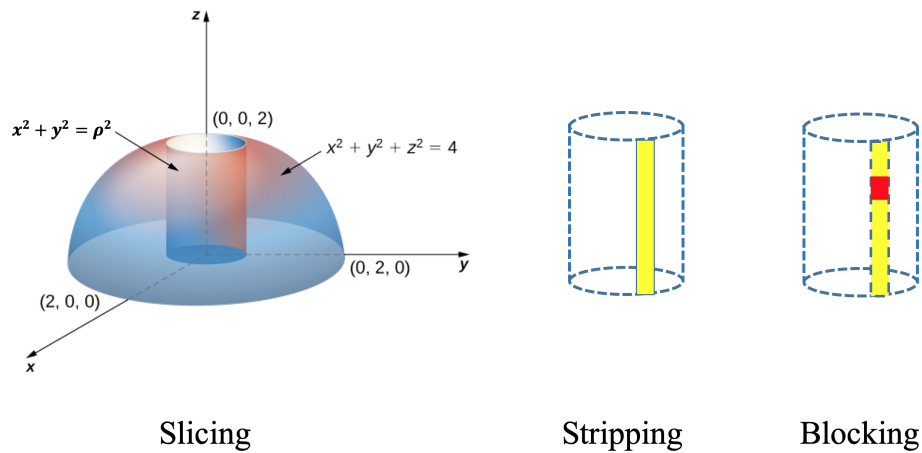
3. Consider a triple integral given in the rectangular coordinates (x, y, z) ,

$$I = \iiint_{\Omega} f(x, y, z) \, dx \, dy \, dz,$$

where the integration Ω is the upper half-ball,

$$\Omega = \{(x, y, z) \mid x^2 + y^2 + z^2 \leq 4, \quad z \geq 0\}.$$

Now let us change the coordinates to the cylindrical coordinate system, (ρ, θ, z) . If the following illustration demonstrates a type of integration strategy,



then determine which of the items below describes integration strategy?

- (a) $\int d\rho \int d\theta \int dz \, f$
- (b) $\int d\theta \int dz \int d\rho \, f$
- (c) $\int dz \int d\rho \int d\theta \, f$
- (d) $\int d\rho \int dz \int d\theta \, f$

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4. A triple integral over a region Ω is defined as

$$I = \iiint_{\Omega} f(x, y, z) dv = \lim_{v_i \rightarrow 0} \sum_{i=1}^n f(x_i, y_i, z_i) \Delta v_i,$$

where $\{\Delta v_1, \dots, \Delta v_i, \dots, \Delta v_n\}$ is a partition of Ω , with $i = 1, \dots, n$, and $(x_i, y_i, z_i) \in \Delta v_i$.

Which of the following statements is/are CORRECT?

- (a) The partition of Ω , $\{\Delta v_1, \dots, \Delta v_i, \dots, \Delta v_n\}$, is arbitrary. A necessary condition for the integral I to exist is the limitation behavior: $\Delta v_i \rightarrow 0$, $\forall i = 1, \dots, n$.
- (b) For this definition of integration there exists a requirement that all the small regions in the partition must be equal to each other, and equal to a standard element Δv ; i.e., $\Delta v_1 = \dots = \Delta v_i = \dots = \Delta v_n = \Delta v$, and the standard Δv goes to 0 in the limitation behavior.
- (c) The integral I is well defined (i.e., having a finite value), even if the integrand $f(x, y, z)$ is unbounded at finite points within the region Ω .
- (d) The integral I must be well defined (i.e., having a finite value), even if the region Ω contains infinitely many singular points.

5. For the Dirac δ -function, if $\omega_0 \neq 0$, compute the integral

$$\int_{-\infty}^{\infty} \cos(\omega t) \delta(\omega - \omega_0) d\omega = \text{_____}, \quad \forall t \in \mathbb{R}.$$

- (a) 0;
- (b) 1;
- (c) $\cos(\omega_0 t)$;
- (d) $\frac{e^{it} + e^{-it}}{2}$.

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SECTION B — GAP-FILLING QUESTIONS

Write your answers on the **Answer Book** provided.

This section is worth a total of **24** marks, with each question worth **4** marks.

6. Evaluate the integral

$$I = \int_0^{+\infty} dx \, e^{-4x^2} = \underline{\hspace{2cm}}.$$

7. Compute the integral

$$\int_{-\pi}^{\pi} \sin\left(3x + \frac{\pi}{4}\right) \sin\left(4x + \frac{\pi}{3}\right) dx = \underline{\hspace{2cm}}.$$

8. The solution to the following equation is $\underline{\hspace{2cm}}$,

$$e^z = 3 + 3i, \quad z \in \mathbb{C}.$$

9. Convert the following integral from the Cartesian coordinates (x, y) to the polar coordinates (ρ, θ) :

$$I = \iint_D x^2 dx dy = \underline{\hspace{2cm}}$$

with $D = \{(x, y) | x^2 + y^2 \leq 4\}$.

(*No need to calculate the value of the integral.)

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10. Evaluate the integral

$$I = \iint_D e^x d\sigma = \underline{\hspace{2cm}},$$

where D is a region enclosed by three lines

$$y = 1, \quad y = 2x, \quad y = 4x.$$

11. Consider a four-dimensional integral in rectangular (Rt) coordinates (x, y, z, w) ,

$$I = \iiint_{\Omega} f(x, y, z, w) dv, \quad \text{where the volume element } dv = dx dy dz dw.$$

Let us change the coordinates to be the four-dimensional spherical coordinates (r, ξ, ϕ, θ) :

$$\begin{cases} x &= [(r \sin \xi) \sin \phi] \cos \theta, \\ y &= [(r \sin \xi) \sin \phi] \sin \theta, \\ z &= (r \sin \xi) \cos \phi, \\ w &= r \cos \xi, \end{cases}$$

where r is the radial coordinate, and ξ , ϕ and θ are angular coordinates. Thus the integral I can be rewritten in the spherical coordinates as

$$I = \iiint_{\Omega} f(r, \xi, \phi, \theta) dv, \quad \text{where the volume element } dv = \underline{\hspace{2cm}}.$$

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SECTION C — EXTENDED ANSWER QUESTIONS

Write your answers on the **Answer Book** provided.

This section is worth a total of **61** marks. The marks of each question are as shown.

- 12. (5 marks)** Evaluate the integral

$$I = \int_0^1 dy \int_y^1 dx e^{-x^2} = \underline{\hspace{2cm}}.$$

- 13. (6 marks)** Use the exponential powers of $\sin \theta$ and $\cos \theta$ to expand $\sin 5\theta = \underline{\hspace{2cm}}$.

- 14. (7 marks)** Consider a polynomial equation

$$p(z) = 4z^5 - 20z^4 + 38z^3 - 28z^2 + 10z = 0, \quad z \in \mathbb{C}.$$

If $z_1 = (2 + i)$ is known to be a root, then try to find the other root(s) of the equation.

- 15. (8 marks)** Let z be a complex number in the complex plane $\mathbb{C}$. For every expression in (a)–(c) below, draw a figure to illustrate the corresponding domain of z .

- (a) $\left\{ z \in \mathbb{C} \mid \operatorname{Im} z > \operatorname{Re} z \right\}, \quad \operatorname{Re} z \text{ — real part of } z, \quad \operatorname{Im} z \text{ — imaginary part of } z.$
- (b) $\left\{ z \in \mathbb{C} \mid |z + 1 + i| + |z - 1 - i| < 6 \right\}.$
- (c) $\left\{ z \in \mathbb{C} \mid z\bar{z} \geq 4 \right\}.$

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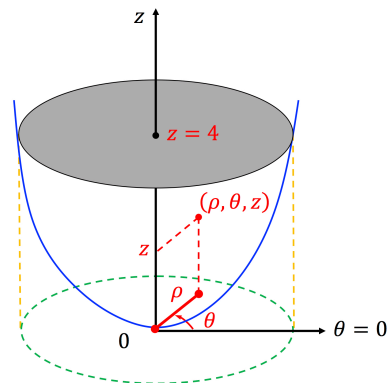
16. (11 marks) Consider a triple integral

$$I = \iiint_{\Omega} z \, dx dy dz,$$

where Ω is the region enclosed by $z = x^2 + y^2$ and $z = 4$.

Change to the coordinate system (ρ, θ, z) , and then use the following two strategies to compute the integral:

- (a) “ $\int dz \int d\theta \int d\rho$ ”; (b) “ $\int d\rho \int dz \int d\theta$ ”.



17. (11 marks) Consider a triple integral

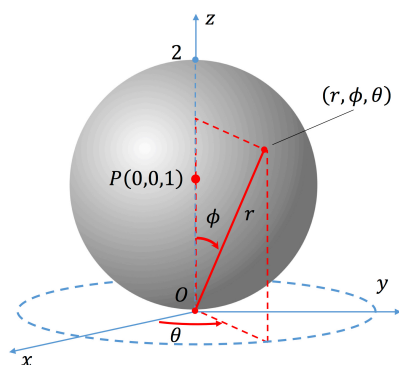
$$I = \iiint_{\Omega} z \, dx dy dz,$$

where Ω is the region enclosed by the sphere centered at $P(0,0,1)$, with radius 1,

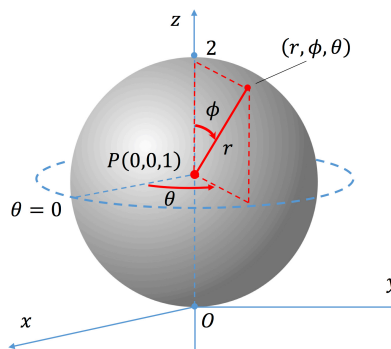
$$x^2 + y^2 + (z - 1)^2 = 1,$$

Compute the integral,

- (a) in terms of a spherical coordinate system (r, ϕ, θ) , with the origin residing at $O(0,0,0)$.
 (b) in terms of another spherical coordinate system (r, ϕ, θ) , with the origin residing at $P(0,0,1)$.



(a)



(b)

BDIC1043J Advanced Mathematics (Module 4; Finance)**18. (13 marks)** (*From Fourier series to Fourier transform*)

The Fourier series for a periodic function is given by

$$f(x) = A_0 + \sum_{n=1}^{\infty} A_n \cos(nx) + \sum_{n=1}^{\infty} B_n \sin(nx), \quad x = \omega t, \quad \omega = \frac{2\pi}{T}, \quad (1)$$

where

$$\begin{aligned} A_0 &= \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx, & A_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx) dx, \\ B_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx) dx. \end{aligned} \quad (2)$$

The Fourier transform for an aperiodic function is given by

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega) e^{i\omega t} d\omega, \quad (3)$$

with

$$G(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt. \quad (4)$$

Give the complete derivation procedure which generalizes the periodic case of Fourier series, eqs.(1)–(2), to the aperiodic case of Fourier transform, eqs.(3)–(4).

Glossary

Angular	角度方向的
Aperiodic function	非周期函数
Arbitrary	任意的
Blocking	切块
Cartesian coordinates	直角坐标, 笛卡尔坐标
Complete derivation procedure	完整推导过程
Complex plane	复平面
Cylindrical coordinate system	柱坐标系
Domain	定义域
Enclose	包围
Exponential power	幂次的
Fourier series	傅立叶级数
Fourier transform	傅立叶变换
Illustration	图示
Infinitely many	无穷多个
Integrand	被积函数
Integration strategy	积分策略
Limitation behavior	极限行为
Low-pass filter	低通过滤
Necessary condition	必要条件
Origin	坐标系原点
Original photo	原始图片
Partition	分割
Periodic function	周期函数
Polar coordinates	极坐标
Polynomial equation	多项式方程
Radial	轴向的, 辐射方向的
Rectangular coordinates	直角坐标
Root	(方程的) 根
Singular point	奇点
Slicing	切片
Spherical coordinate system	球坐标系
Standard element	标准积分微元
Stripping	切条
Triple integral	三重积分
Unbounded	无界
Upper half-ball	上半球
Volume element	体积元
Well defined	良好定义的