



北京工业大学北京 – 都柏林国际学院
BEIJING-DUBLIN INTERNATIONAL COLLEGE AT BJUT

SEMESTER 2 EXAMINATIONS

ACADEMIC YEAR 2022/2023

EEEN3004J Digital Signal Processing

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BJUT Student ID: _____ **UCD Student ID:** _____

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge: _____ **(Signature)**

Time Allowed: 2 HOURS

Instructions for Candidates

This examination paper contains **FOUR** questions.

Answer any **THREE** questions for a maximum of 300 marks.

All four questions carry equal marks.

The distribution of marks for the parts of each question is indicated in () brackets

Instructions for Invigilators

Non-programmable calculators are permitted.

No rough-work paper is to be provided for candidates.

Formulas

These formulas are provided without further context.

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{-j \frac{n 2\pi t}{T}}$$

$$C_n = \frac{1}{T} \int_{-T/2}^{T/2} T/2 f(t) e^{-j \frac{n 2\pi t}{T}} dt$$

$$F(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

$$F(s) = \int_{-\infty}^{\infty} f(t) e^{-st} dt$$

$$F(z) = \sum_{n=-\infty}^{\infty} f_n z^{-n}$$

$$h(t) = f * g = \int_{-\infty}^{\infty} f(\tau) g(t - \tau) d\tau$$

$$h(t) = f * g \Leftrightarrow H(j\omega) = F(j\omega)G(j\omega)$$

$$f(t) = \begin{cases} 1, & \text{if } |t| < \frac{L}{2} \\ 0, & \text{otherwise} \end{cases} \Leftrightarrow F(j\omega) = L \operatorname{sinc}\left(\frac{\omega L}{2}\right) = L \frac{\sin\left(\frac{\pi \omega L/2}{2}\right)}{\frac{\pi \omega L}{2}}$$

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$$

$$y_n = \sum_{k=0}^M b_k x_{n-k} - \sum_{l=1}^L a_l y_{n-l}$$

$$\sum_{n=0}^{N-1} e^{j \frac{2\pi n k}{N}} = \begin{cases} N, & \text{if } k \text{ is an integer multiple of } N \\ 0, & \text{otherwise} \end{cases}$$

$$\cos(60^\circ) = \frac{1}{2}$$

$$\sin(60^\circ) = \frac{\sqrt{3}}{2}$$

The variance of a uniform random variable on $\pm\Delta/2$ is $\Delta^2/12$

Q 1

a. State the definitions of each of the following.

- i The discrete-time Fourier transform.
- ii The discrete Fourier transform.
- iii The z-transform.
- iv Discrete periodic convolution.

(20 marks)

b.

- i Prove that the z-transform is linear.
- ii If the z-transform of a discrete signal, x_n , is $X(z)$, derive the z-transform of x_{n-K} .

(30 marks)

c.

- i Calculate the z-transform of the unit step function, u_n .
- ii Calculate the z-transform of $f_n = \frac{1}{2}^n$.
- iii Hence or otherwise find the discrete-time Fourier transform of $f_n = u(nT) \star \frac{1}{2}^{(n-1)T}$, for $T = 3$ seconds and $n \in \mathbb{Z}$. The symbol $\star$ denotes direct discrete convolution. You may ignore the ROC.

Show your work.

(30 marks)

d. Why do we use the z-transform to describe the behaviour of a digital filter? Why not use the Fourier transform instead?

(20 marks)

(Total: 100 marks)

Q 2

a.

- State the difference equation that describes a general digital filter.
- Define the transfer function of a digital filter, $\tilde{H}(z)$, in terms of the filter input and output, and in terms of the filter coefficients. It is not necessary to derive this.

Be sure to define every term in the equations. (20 marks)

b. Starting with the difference equation, prove that the transfer function of a digital filter is given by

$$\tilde{H}(z) = b_0 z^{L-M} \frac{\prod_{m=0}^M (z - z_m)}{\prod_{l=1}^L (z - p_l)}.$$

(30 marks)

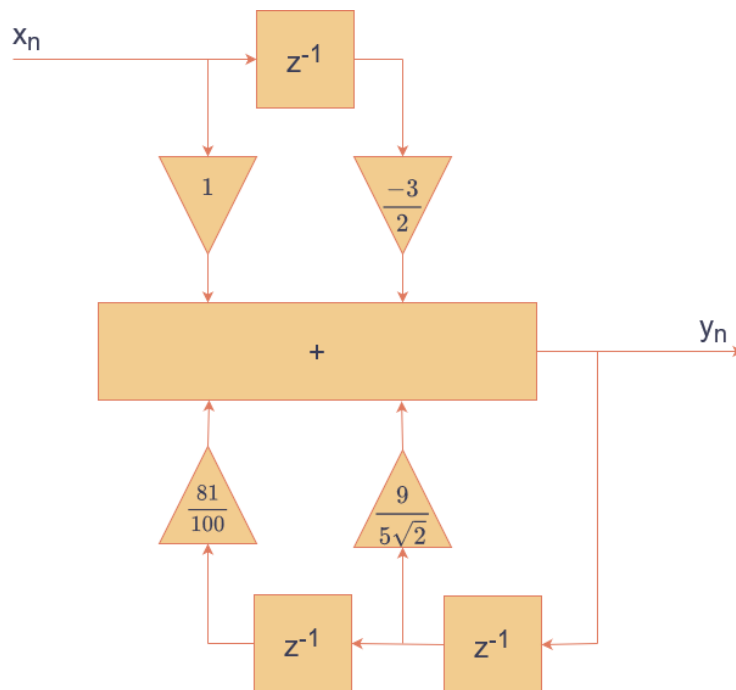


Figure 1: Digital filter for Q2(c) and (d).

c. Given the filter shown in Fig. 1, answer the following.

- What is the difference equation for this filter?
- Derive its transfer function.
- Draw the pole-zero plot for this filter.

(30 marks)

d.

- i Determine the stability of the filter from part (c). Explain your answer: an unsupported answer will not receive marks.
- ii Sketch the amplitude response of the filter from part (c), based on the pole-zero plot.

(20 marks)

(Total: 100 marks)

Q 3

a.

- i Define the Dirac delta function in time.
- ii Sketch it in the time domain.
- iii Define the sifting property of the delta function.
- iv Define a Dirac delta train.

(20 marks)

b. An arbitrary signal, $f(t)$, has Fourier transform $F(j\omega)$. The signal is sampled at regular intervals with sampling interval T . Derive an expression for the spectrum of the sampled signal. (20 marks)

c. The spectrum, $F(j\omega)$, of a given signal is non-zero only on the range $-B/2 \leq \omega \leq B/2$.

- i Sketch the spectrum of the signal after it has been sampled with interval $T = 4B/5$. Label your axes clearly.
- ii Explain the significance of your plot for how we choose the sampling rate when sampling a signal.

(30 marks)

- d.** Given the signal $f(t) = \cos(10\pi t)$, answer the following.
- i Determine a suitable sampling period T according to Nyquist's theorem.
 - ii Calculate the first four samples, i.e. at $t \in \{0, T, 2T, 3T\}$.
 - iii Calculate the spectrum (i.e. the DFT) of this sampled signal.
 - iv Plot the amplitude spectrum. Label your axes.

(30 marks)

(Total: 100 marks)**Q 4****a.**

- i Why is an FIR filter called an FIR filter? Give an example.
- ii Draw an example pole-zero diagram for a real valued low pass FIR filter containing exactly 5 zeros.
- iii What is a linear phase filter and why are they important?

(30 marks)

b. Prove that an odd-length real filter with anti-symmetric coefficients is a linear phase filter. (20 marks)

c. If an FIR filter has a pair of zeros at positions $z = \pm 1$

- i Draw a pole-zero diagram for this filter.
- ii Write its transfer function.
- iii Write its time domain description for a causal version of this filter.

(30 marks)

d.

- i Sketch the transfer function of the filter in part (c).
- ii What kind of filter is it?

(20 marks)

(Total: 100 marks)

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