北京工业大学北京－都柏林国际学院
BEIJING-DUBLIN INTERNATIONAL COLLEGE AT BJUT

SEMESTER 2 EXAMINATIONS

ACADEMIC YEAR 2023/2024

EEEN3004J Digital Signal Processing

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BJUT Student ID: _____ UCD Student ID: _____

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge: _____(Signature)

Time Allowed: 2 HOURS

Instructions for Candidates

This examination paper contains **FOUR** questions.
Answer any **THREE** questions for a maximum of 300 marks.
All four questions carry equal marks.
The distribution of marks for the parts of each question is indicated in ( ) brackets

Instructions for Invigilators

Non-programmable calculators are permitted.
No rough-work paper is to be provided for candidates.

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Formulas

These formulas are provided without further context.

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{-j\frac{n2\pi t}{T}}$$

$$C_n = \frac{1}{T}\int_{-T/2} T/2 f(t) e^{-j\frac{n2\pi t}{T}} dt$$

$$F(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

$$F(s) = \int_{-\infty}^{\infty} f(t) e^{-st} dt$$

$$F(z) = \sum_{n=-\infty}^{\infty} f_n z^{-n}$$

$$h(t) = f * g = \int_{-\infty}^{\infty} f(\tau) g(t-\tau) d\tau$$

$$h(t) = f * g \Leftrightarrow H(j\omega) = F(j\omega)G(j\omega)$$

$$f(t) = \begin{cases} 1, & \text{if } |t| < \frac{L}{2} \\ 0, & \text{otherwise} \end{cases} \Leftrightarrow F(j\omega) = L\text{sinc}\left(\frac{\omega L}{2}\right) = L\frac{\sin\left(\frac{\pi\omega L/2}{2}\right)}{\frac{\pi\omega L}{2}}$$

$$\int_{-\infty}^{\infty} |x(t)|^2 = \int_{-\infty}^{\infty} |X(j\omega)|^2$$

$$y_n = \sum_{k=0}^{M} b_k x_{n-k} - \sum_{l=1}^{L} a_l y_{n-l}$$

$$\sum_{n=0}^{N-1} e^{j\frac{2\pi nk}{N}} = \begin{cases} N, & \text{if } k \text{ is an integer multiple of } N \\ 0, & \text{otherwise} \end{cases}$$

$$\cos(60^\circ) = \frac{1}{2}$$

$$\sin(60^\circ) = \frac{\sqrt{3}}{2}$$

The variance of a uniform random variable on $\pm\Delta/2$ is $\Delta^2/12$

Q 1

a. Define each of the following mathematically.

i A Dirac delta function.

ii An impulse train.

iii The Discrete Fourier Transform.

(15 marks)

b.

i Derive the spectrum of an impulse train.

ii Hence derive the spectrum of a sampled signal.

(30 marks)

c.

i State Nyquist's theorem.

ii Explain the significance of your answer to question 1.b.ii. for Nyquist's theorem.

(25 marks)

d.

i Calculate the elements of a 3-point DFT matrix. Show your work.

ii Hence or otherwise calculate the DFT of the sequence $f = \{3, 1, 2\}$. If you are unable to perform the calculation yourself, partial marks are available for the MATLAB code to do the same calculation instead.

iii That sequence, f, was measured with a sampling frequency of 10 kHz. Sketch the spectrum you calculated with the correct frequency axis.

(30 marks)

(Total: 100 marks)

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Q 2

a. State Parseval's theorem for discrete-time sequences. (20 marks)

b. State and prove the Fourier dual property of direct discrete convolution. (26 marks)

c. Derive a radix-3 decimation-in-time FFT algorithm. (30 marks)

d. The signal f_n is 16 samples of a 1Hz cosine, sampled at 2 Hz. The signal is corrupted by a little additive white Gaussian noise. The DFT of this signal is F_m. If we break f_n into four pieces of equal length, we can calculate the DFT of each of those pieces. We then add the resulting 4-point DFTs together to make $\hat{F}_m$.

i Sketch F_m, taking care to label the frequency axis correctly.

ii Sketch $\hat{F}_m$, again taking care to label the frequency axis.

iii Explain the differences between these two spectra.

(24 marks)

(Total: 100 marks)

Q 3

A digital filter has the following difference equation

$$y[n] = 0.2929x[n] + 0.5858x[n-1] + 0.2929x[n-2] - 0.1716y[n-1] - 0.3689y[n-2] \quad (1)$$

Analyse this filter using techniques you have learned in this module. You should draw conclusions about the nature of the filter.

(Total: 100 marks)

Q 4

A digital filter is to meet the specification given in Fig. 1. The parameters are as follows.

f_{passband}	0.4
f_{stopband}	0.6
A_s	-20 dB
Maximum passband ripple	10 dB

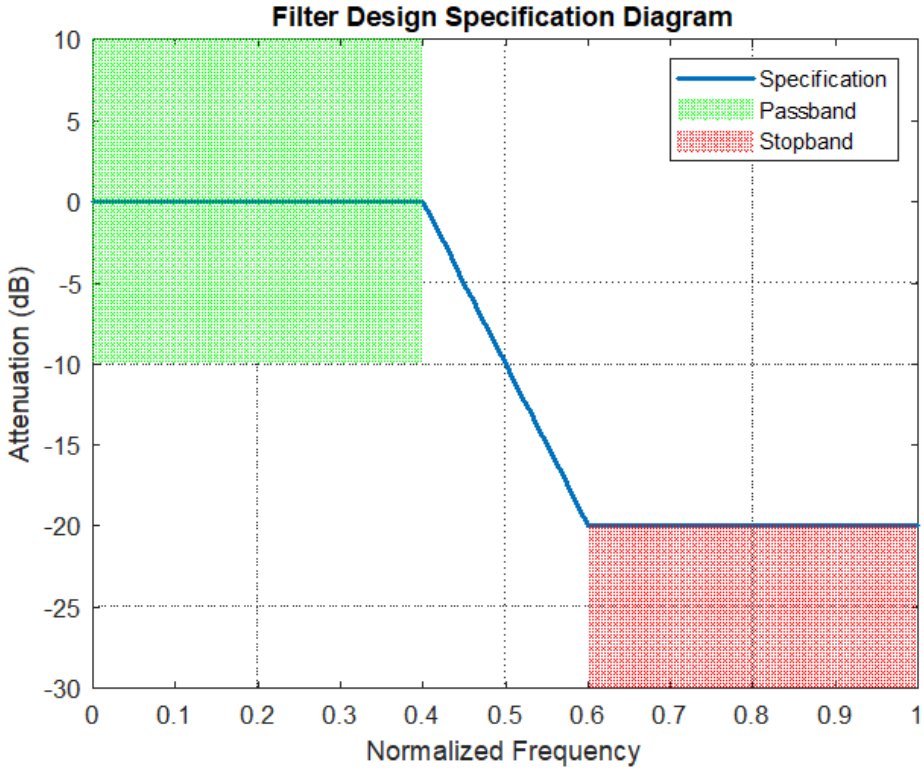

Figure 1: Filter specification.

i Outline the steps you would take to design this filter by hand. Partial marks are available for explaining how to design it with MATLAB instead.

ii Calculate the coefficients you get for a 3-tap FIR filter or a 2rd order IIR filter.

(Total: 100 marks)

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