



北京工业大学北京 – 都柏林国际学院
BEIJING-DUBLIN INTERNATIONAL COLLEGE AT BJUT

SEMESTER 2 RESIT EXAMINATIONS
ACADEMIC YEAR 2021/2022

EEEN3004J Digital Signal Processing

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BJUT Student ID: _____ **UCD Student ID:** _____

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge: _____ **(Signature)**

Time Allowed: 2 HOURS

Instructions for Candidates

This examination paper contains **FOUR** questions.

Answer any **THREE** questions for a maximum of 300 marks.

All four questions carry equal marks.

The distribution of marks for the parts of each question is indicated in () brackets

Instructions for Invigilators

Non-programmable calculators are permitted.

No rough-work paper is to be provided for candidates.

Formulas

These formulas are provided without further context.

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{-j \frac{n 2\pi t}{T}}$$

$$C_n = \frac{1}{T} \int_{-T/2}^{T/2} T/2 f(t) e^{-j \frac{n 2\pi t}{T}} dt$$

$$F(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

$$F(s) = \int_{-\infty}^{\infty} f(t) e^{-st} dt$$

$$F(z) = \sum_{n=-\infty}^{\infty} f_n z^{-n}$$

$$h(t) = f * g = \int_{-\infty}^{\infty} f(\tau) g(t - \tau) d\tau$$

$$h(t) = f * g \Leftrightarrow H(j\omega) = F(j\omega)G(j\omega)$$

$$f(t) = \begin{cases} 1, & \text{if } |t| < \frac{L}{2} \\ 0, & \text{otherwise} \end{cases} \Leftrightarrow F(j\omega) = L \operatorname{sinc}\left(\frac{\omega L}{2}\right) = L \frac{\sin\left(\frac{\pi \omega L/2}{2}\right)}{\frac{\pi \omega L}{2}}$$

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$$

$$y_n = \sum_{k=0}^M b_k x_{n-k} - \sum_{l=1}^L a_l y_{n-l}$$

$$\sum_{n=0}^{N-1} e^{j \frac{2\pi n k}{N}} = \begin{cases} N, & \text{if } k \text{ is an integer multiple of } N \\ 0, & \text{otherwise} \end{cases}$$

$$\cos(60^\circ) = \frac{1}{2}$$

$$\sin(60^\circ) = \frac{\sqrt{3}}{2}$$

The variance of a uniform random variable on $\pm\Delta/2$ is $\Delta^2/12$

Q 1



Figure 1: A fingertip pulse oximeter uses PPG to measure heart rate.

a. Photoplethysmography (PPG) is a technique that measures relative blood volume changes in the blood vessels close to the skin. It can be used to detect a person's heart rate non-invasively. In a given PPG system, the sensor output is passed through an anti-aliasing filter before being sampled at 100 Hz.

- i What is aliasing and how does an anti-aliasing filter avoid it?
- ii Is this a digital or an analogue filter? Explain your answer.

(20 marks)

b. What specifications should this particular anti-aliasing filter have? (15 marks)

c. After the signal has been sampled, a further filter is used to remove noise and unwanted signal components. That filter is to be a low-pass FIR filter with unit pass-band gain, and a 20 Hz cut-off frequency. The filter is to be limited to 7 taps with real coefficients. By any means you have learned, design this filter. (35 marks)

d. To simplify implementation, we required that the FIR filter in the previous section had real coefficients.

- i When an FIR filter has real coefficients, what can we say about it in the frequency domain?
- ii Prove the property you stated in (i).

(30 marks)

(Total: 100 marks)

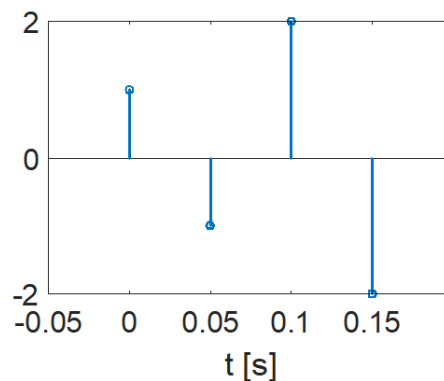
Q 2**a.**

- i Define the discrete time Fourier transform (DTFT).
- ii State the shift theorem for the DTFT.

(30 marks)

b. Prove the shift theorem for the DTFT.

(20 marks)

Figure 2: Signal $\{f_n\}$ for part (c).

c. Calculate the DFT of the signal $\{f_n\} = \{1, -1, 2, -2\}$. Show your work. If the sampling period of this signal is $T = 0.05$ s, plot the spectrum with a labelled frequency axis.

(30 marks)

d. Consider the following three discrete signals.

$$\{f_n\} = \{1, 2j, 2, -j, 1, 5\}$$

$$\{g_n\} = \{0, 0, 0, 1, 2j, 2, -j, 1, 5, 0, 0, 0\}$$

$$\{h_n\} = \{1, 0, 2j, 0, 2, 0, -j, 0, 1, 0, 5, 0\}$$

- i Explain the relationship between the spectra of $\{f_n\}$ and $\{g_n\}$.
- ii Explain the relationship between the spectra of $\{f_n\}$ and $\{h_n\}$.

(20 marks)

(Total: 100 marks)

Q 3**a.**

- i Why is an FIR filter called an FIR filter? Give an example.
- ii Draw an example pole-zero diagram for a real valued low pass filter containing exactly 5 zeros.
- iii What is a linear phase filter and why are they important?

(30 marks)

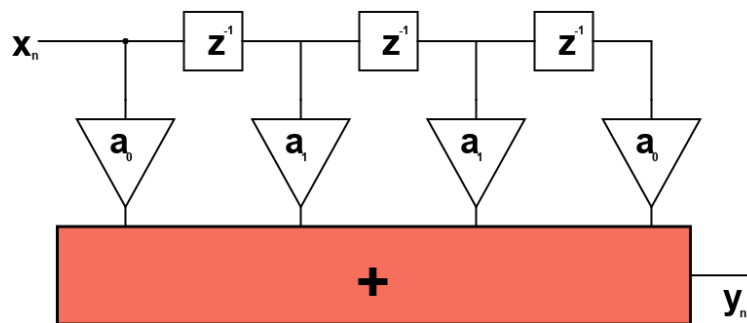


Figure 3: An FIR Filter with symmetric coefficients.

b. Prove that an even-length real filter with symmetric coefficients is a linear phase filter. (20 marks)

c. If an FIR filter has a pair of zeros at position $\pm j$

- i Draw a pole-zero diagram for this filter.
- ii Write its transfer function.
- iii Write its time domain description for a causal version of this filter.

(30 marks)

d.

- i Sketch the transfer function of the filter in part (c).
- ii What kind of filter is it?

(20 marks)

(Total: 100 marks)

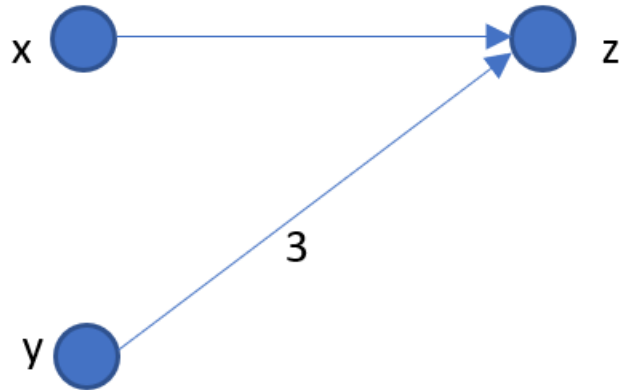
Q 4

Figure 4: A signal flow diagram.

a. Consider the signal flow diagram in Fig. 4. Assuming this is part of an FFT of size N , write an expression for z in terms of x and y . (20 marks)

b. Prove the following.

i $W_N^n = -W_N^{n-N/2}$

ii $W_N^{nm} = W_{N/m}^n$

iii If a sequence $\{f_n\}$ has DFT $\{\bar{F}_m\}$, the sequence $\{a_n\} = \{f_{2n}\}$ has DFT $\{\bar{A}_m\}$, and the sequence $\{b_n\} = \{f_{2n+1}\}$ has DFT $\{\bar{B}_m\}$, prove that $\bar{F}_m = \bar{A}_m + W_N^m \bar{B}_m$.

(30 marks)

c. Using the results above, find the formula for a single stage of decomposition in a radix-2 decimation-in-time fast Fourier transform algorithm. (25 marks)

d. Using a size 4 DFT matrix and the formula derived above to perform a single stage of decomposition, calculate the DFT of the signal $\{f_n\} = \{1, 2.2, 1.1, 0.3, 1.1, 2.1, 1.2, 0\}$. Show your workings. (25 marks)

(Total: 100 marks)

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