



北京工业大学北京 – 都柏林国际学院
BEIJING-DUBLIN INTERNATIONAL COLLEGE AT BJUT

SEMESTER 2 RESIT EXAMINATIONS
ACADEMIC YEAR 2023/2024

EEEN3004J Digital Signal Processing

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BJUT Student ID: _____ **UCD Student ID:** _____

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge: _____ **(Signature)**

Time Allowed: 2 HOURS

Instructions for Candidates

This examination paper contains **FOUR** questions.

Answer any **THREE** questions for a maximum of 300 marks.

All four questions carry equal marks.

The distribution of marks for the parts of each question is indicated in () brackets

Instructions for Invigilators

Non-programmable calculators are permitted.

No rough-work paper is to be provided for candidates.

Formulas

These formulas are provided without further context.

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{-j \frac{n2\pi t}{T}}$$

$$C_n = \frac{1}{T} \int_{-T/2}^{T/2} f(t) e^{-j \frac{n2\pi t}{T}} dt$$

$$F(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

$$F(s) = \int_{-\infty}^{\infty} f(t) e^{-st} dt$$

$$F(z) = \sum_{n=-\infty}^{\infty} f_n z^{-n}$$

$$h(t) = f * g = \int_{-\infty}^{\infty} f(\tau) g(t - \tau) d\tau$$

$$h(t) = f * g \Leftrightarrow H(j\omega) = F(j\omega)G(j\omega)$$

$$f(t) = \begin{cases} 1, & \text{if } |t| < \frac{L}{2} \\ 0, & \text{otherwise} \end{cases} \Leftrightarrow F(j\omega) = L \operatorname{sinc} \left(\frac{\omega L}{2} \right) = L \frac{\sin \left(\frac{\pi \omega L / 2}{2} \right)}{\frac{\pi \omega L}{2}}$$

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$$

$$y_n = \sum_{k=0}^M b_k x_{n-k} - \sum_{l=1}^L a_l y_{n-l}$$

$$\sum_{n=0}^{N-1} e^{j \frac{2\pi n k}{N}} = \begin{cases} N, & \text{if } k \text{ is an integer multiple of } N \\ 0, & \text{otherwise} \end{cases}$$

$$\cos(60^\circ) = \frac{1}{2}$$

$$\sin(60^\circ) = \frac{\sqrt{3}}{2}$$

The variance of a uniform random variable on $\pm\Delta/2$ is $\Delta^2/12$

Q 1

a. Which ONE of the following best defines a Dirac Delta function.

i $\delta(t) = \begin{cases} 1, & n = 0 \\ 0, & \text{otherwise} \end{cases}$

ii $\delta(t) = \begin{cases} \infty, & n = 0 \\ 0, & \text{otherwise} \end{cases}$

iii $\int_D \delta(t) dt = \begin{cases} 1, & t = 0 \\ 0, & \text{otherwise} \end{cases}$

iv $\int_D \delta(t) dt = \begin{cases} 1, & t = 0 \in D \\ 0, & \text{otherwise} \end{cases}$

(15 marks)

b. Which ONE of the following best defines a discrete Fourier transform.

i $f(t) = \sum_{n=-\infty}^{\infty} C_n e^{-j \frac{n2\pi t}{T}}$

ii $F(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$

iii $\bar{F}(j\omega) = \sum_{n=-\infty}^{\infty} f(nT) e^{-j\omega nT}$

iv $\bar{F}_m = \sum_{n=0}^{N-1} f_n e^{-j2\pi nm/N}$

(15 marks)

c. A continuous signal, $f(t)$, is sampled at a rate $f_s = 1/T$ Hz, giving the discrete signal $\bar{f}(t) = \sum_{n=-\infty}^{\infty} f(t) \delta(t - nT)$, $n \in \mathbb{Z}$.

Using the sifting property of the Dirac delta function, derive Fourier transform of $\bar{f}(t)$ in terms of f_n , the samples of $f(t)$ (i.e. derive the formula for the DTFT). (30 marks)

d. Fig. 1 shows an audio spectrum for a recorded piece of music. Estimate the sampling period of this signal. Show your work. (20 marks)

e. A recording engineer proposes reducing the sampling period of this recording by 25%. What would that change about the signal? Your answer should consider (i) file size, (ii) accuracy of the recording, and (iii) how to process the signal to limit the problems caused by this change. Be specific. (20 marks)

(Total: 100 marks)

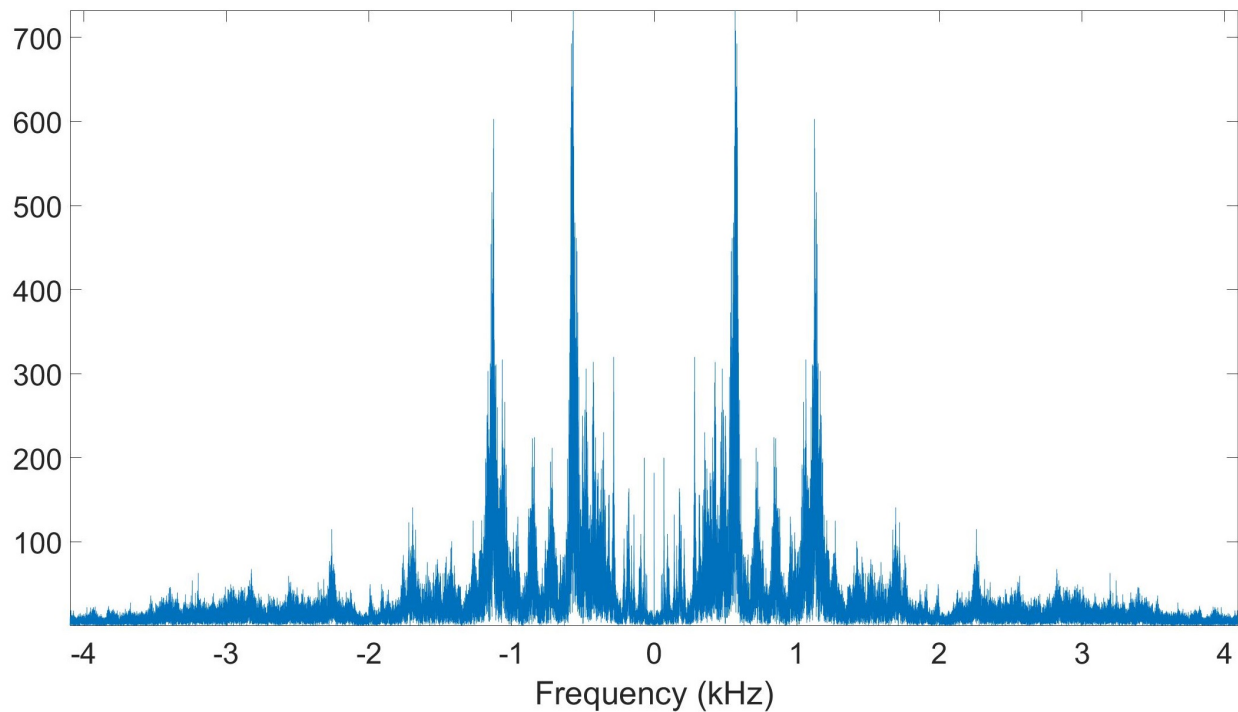


Figure 1: Audio spectrum for question 1.

Q 2

a. What is the relationship between the DFT and the FFT? (10 marks)

b. In terms of the number of samples, N , and using big O notation,

i what is the order of complexity of the DFT?

ii what is the order of complexity of the FFT?

(20 marks)

c. In this module, we derived a radix 2 FFT algorithm. This can split an N -point DFT calculation into two $N/2$ -point DFTs. If $N = 2^q$, we can do q stages of decomposition. How can we use a radix 2 FFT algorithm to calculate the spectrum of a signal with 249 samples? (10 marks)

d.

i Given the radix 3 FFT formula, $F_m = A_m + W_N^m B_m + W_N^{2m} C_m$, draw the butterfly diagram for one stage of decomposition of a 6-point FFT.

ii Hence or otherwise, calculate the DFT of the signal $\{1, 1, 1, 1, 0, -1\}$

(40 marks)

e. Explain what is happening in Eq. 1.

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{pmatrix} \begin{pmatrix} f_0 \\ f_1 \\ f_2 \\ f_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -j \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & j \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} f_0 \\ f_1 \\ f_2 \\ f_3 \end{pmatrix} \quad (1)$$

(20 marks)

(Total: 100 marks)

Q 3

a. Copy the following sentences into your answer-book, filling in the blanks. Each one is a statement about interpreting pole-zero plots.

i A digital filter with no poles is called a/an _____ filter.

ii A digital filter with poles and zeros which are symmetrical around the real z axis has _____ coefficients.

iii A digital filter with all of its poles _____ is BIBO stable.

iv A digital filter with a zero on the unit circle has a frequency response with _____.

v An FIR filter with zeros in invert pairs (i.e. if ζ is a root, so too is ζ^{-1}) is a _____ filter.

(35 marks)

b. The pole zero plot of a 4th order digital Chebyshev filter is given in Fig. 2. Which of the four frequency responses in Fig. 3 belongs to that filter? Explain your reasoning. (25 marks)

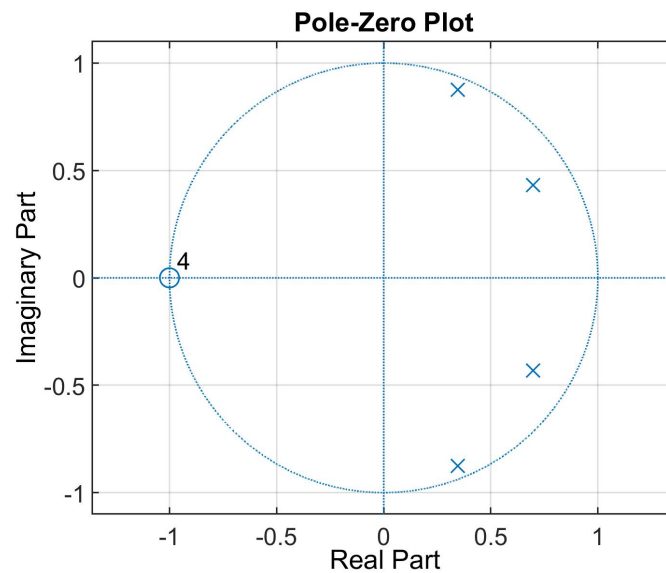


Figure 2: Pole zero plot for question 3.

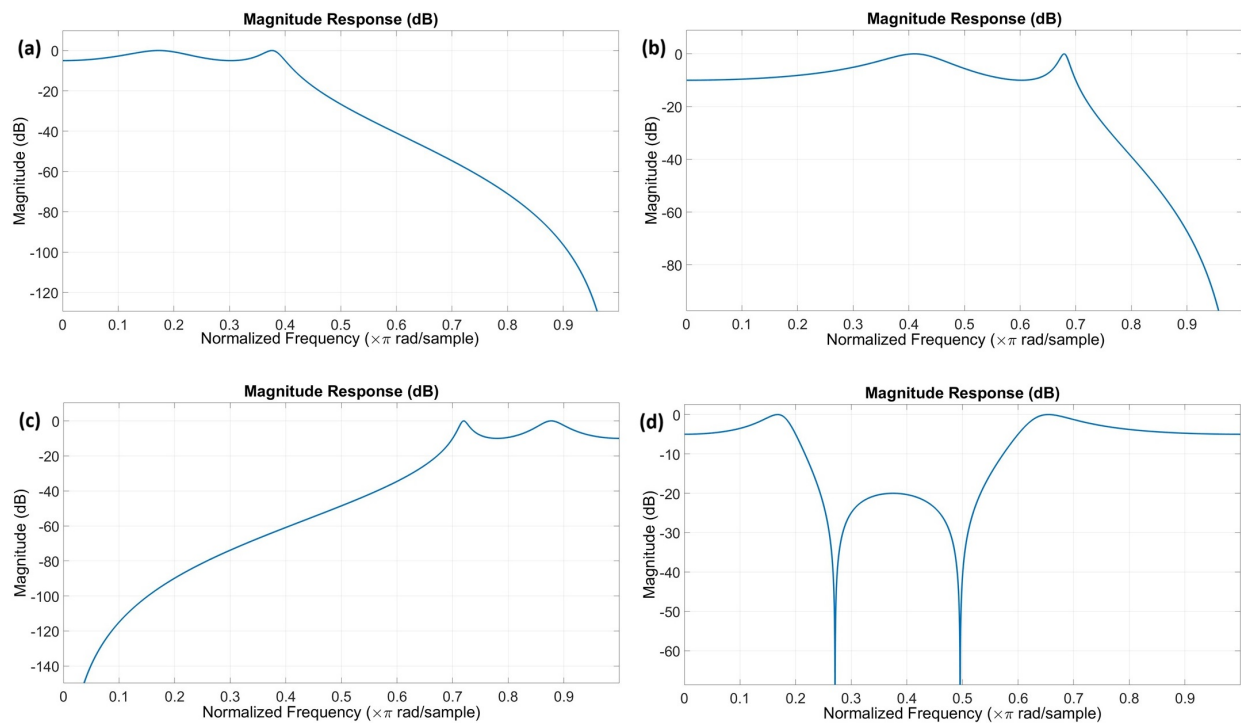


Figure 3: Filter responses for question 3.

c. A Chebyshev filter is a kind of equiripple filter.

i What does equiripple refer to?

ii Equiripple filters minimise what per the Chebyshev equi-oscillation theorem?

(20 marks)

- d. Let $\{h_j\}$ for $-\infty < j < \infty$ be the impulse response of a digital filter. Prove that a necessary and sufficient condition for stability is that $\sum_{j=-\infty}^{\infty} |h_j| < \infty$. (20 marks)

(Total: 100 marks)

Q 4

- a. If the z-transform of x_n is $\tilde{X}(z)$, what is the z-transform of x_{n-2} ? (20 marks)
- b. Prove that the digital filter of Eq. 2 is linear.

$$y_n = \alpha x_n + (1 - \alpha)y_{n-1} \quad (2)$$

(25 marks)

- c. Find the transfer function of the filter defined by Eq. 2. (25 marks)
- d. A digital filter has the following transfer function.

$$\tilde{H}(z) = z^{-1} \frac{(z - 1)(z^2 - 0.25)}{z^2 + 0.25} \quad (3)$$

- i How many non-trivial poles and zeros does this filter have?
- ii Calculate the poles and zeros of the filter.
- iii Sketch the frequency response of this filter.

(30 marks)

(Total: 100 marks)

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