北京工业大学北京－都柏林国际学院
BEIJING-DUBLIN INTERNATIONAL COLLEGE AT BJUT

SEMESTER 2 FINAL EXAMINATIONS

ACADEMIC YEAR 2022/2023

EEEN3018J Numerical Methods and Optimization

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I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge: _____(Signature)

Time Allowed: 2 HOURS

Instructions for Candidates

This examination paper contains **FIVE** questions.
Answer question 1 **AND EITHER** question 2 or question 3, **AND EITHER** question 4 or question 5 for a maximum of 200 marks.
Questions 2 and 3 carry equal marks. Questions 4 and 5 carry equal marks.
The distribution of marks for the parts of each question is indicated in ( ) brackets

Instructions for Invigilators

Non-programmable calculators are permitted.
No rough-work paper is to be provided for candidates.

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Formulas

These formulas are provided without further context.

$f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + ...$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_{n+1} = x_n - \frac{(x_n - x_{n-1})f(x_n)}{f(x_n) - f(x_{n-1}}$

$$\begin{pmatrix} \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i & n \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{n} x_i y_i \\ \sum_{i=1}^{n} y_i \end{pmatrix}$$

$A^T A c = A^T y$

$a_i = \frac{y_{i+1} - y_i}{x_{i+1} - x_i}$

$b_i = y_i$

$$\begin{bmatrix} \frac{2h_0}{3} & \frac{h_0}{3} & 0 & \cdots & 0 & 0 & 0 \\ \frac{h_0}{3} & \frac{2}{3}(h_1 + h_0) & \frac{h_1}{3} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \frac{h_{n-2}}{3} & \frac{2}{3}(h_{n-2} + h_{n-1}) & \frac{h_n}{3} \\ 0 & 0 & 0 & \cdots & 0 & \frac{h_{n-1}}{3} & \frac{2h_{n-1}}{3} \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_{n-1} \\ b_n \end{bmatrix} = \begin{bmatrix} \frac{f_0}{h_0} - \alpha \\ \frac{f_1}{h_1} - \frac{f_0}{h_0} \\ \vdots \\ \frac{f_{n-1}}{h_{n-1}} - \frac{f_{n-2}}{h_{n-2}} \\ \beta - \frac{f_{n-1}}{h_{n-1}} \end{bmatrix}$$

$hf(\frac{h}{2})$

$h[f(0) + f(h)]$

$\frac{h}{3}[f(0) + 4f(h) + f(2h)]$

$x_{n+1} = x(t_n) + \Delta t f(t_n, x_n)$

$x_{n+1} = x_n + \frac{1}{2}\Delta t[f(t_n, x_n) + f(t_n + \Delta t, x_{n+1}^p)]$

$k_1 = \Delta t f(t_n, x_n)$

$k_2 = \Delta t f(t_n + \alpha \Delta t, x_n + \beta k_1)$

$x_{n+1} = x_n + a k_1 + b k_2$

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Q 1

a. Convert the following program to Standard Equality Form:

$$\begin{aligned} \min \quad & z(x) = 3x1 + 2x2 + x3 \\ \text{s.t.} \quad & 3x1 + x2 && \geq 7 \\ & 3x2 + 2x3 && \leq 9 \\ & 2x1 + 2x2 + 2x3 && = -7 \\ & x1, x2 && \geq 0 \end{aligned}$$

(10 marks)

b. A company wishes to produce chairs and tables. Each week the company purchases wooden boards at €5 per meter for furniture and can buy up to 450 hours of labour at €14 per hour. It knows that to make a chair requires 5 meters of board and 5 labour-hours and can sell for a price of €120, while each table uses 20 meters of board and 8 labour-hours and can sell for €350. The company sells the furniture in sets, so must produce at least 2 chairs for every 1 table produced.
Write a problem formulation to maximise the weekly profits of the company while considering the constraints of the system.

(10 marks)

c. Find the Dual (D) for the Primal program (P) below:

$$\begin{aligned} \min \quad & -2x1 + 2x2 + x3 \\ \text{s.t.} \quad & -2x1 + x2 - 2x3 && \geq 8 \\ & 3x2 + 6x3 && \geq -4 \\ & x1, x2 && \geq 0 \end{aligned}$$

consider the solution $x = \{0, \frac{31}{3}, \frac{-21}{3}\}$, $y = \{\frac{11}{4}, \frac{7}{12}\}$. Are the solutions feasible and optimal for the Primal – Dual pair?

(10 marks)

d. Consider the LP problem:

$$\max\{c^{\mathsf{T}}x : Ax = b, x \geq 0\}, where$$

$$A = \begin{pmatrix} 3 & 2 & -4 & 2 \\ 1 & 3 & 2 & 3 \end{pmatrix} \quad b = \begin{pmatrix} 4 \\ 6 \end{pmatrix} \quad c^{\mathsf{T}} = (\,-2, 3, 4, -3) \quad B = \{2, 3\}$$

write the LP problem in canonical form for the Basis B.

(10 marks)

e. From the given function:

$$f(x) = 3x_3{}^2 + 2x_2{}^2 + 4x_3x_2 + 9x_3x_1 - 3x_2x_1 + 4x_3 - 2x_2 - 7x_1 + 4$$

Determine if the function is strictly convex.
(Hint: first determine the Hessian matrix(H), $\nabla^2 F$)

$(10\ marks)$

(Total: 50 marks)

Q 2

From the following optimisation problem:

$$\begin{aligned} \max \quad & z(x) = (1, 2, 0, 0)x \\ \text{s.t.} \quad & \begin{pmatrix} 2 & 1 & 1 & 0 \\ -1 & 3 & 0 & 1 \end{pmatrix} x \begin{matrix} \leq \\ \leq \end{matrix} \begin{pmatrix} 8 \\ 3 \end{pmatrix} \\ & x \geq 0 \end{aligned}$$

Use the simplex algorithm to find the optimal solution x* and optimal value z(x*) for the problem.
- Use Bland's rule to decide new entries to the Basis set.

(Total: 50 marks)

Q 3

For the following function:

$$f(x, y) = 5x^2 + 2y^2 + 2xy - 4x + 9y - 15$$

Find the point [x,y]' derived after 2 iterations using Gradient Decent - Steepest decent search from the point:

$$[x, y] = [5, 5]$$

What is the location of this point [x,y]'?
what is the function value at the point [x,y]'?
Give your answer up to 3 decimal places.

Note:
For each line search work out the minimum point analytically.

(Total: 50 marks)

Q 4

a. Derive the formula for the function $y = g_i(x)$ that linearly interpolates the endpoints (x_i, y_i) and (x_{i+1}, y_{i+1}). (30 marks)

b. A wearable electro-encephalogram (EEG) monitor samples the EEG using a level-crossing analog-to-digital converter to reduce power consumption. At time indices $t = 1.45$ s and $t = 1.4768$ s, the monitor returns voltages of 0.9276 mV and 1.015625 mV respectively. Estimate the voltage at $t = 1.46$ s using linear interpolation. (30 marks)

c. Compare and contrast linear interpolation and cubic interpolation: what are the advantages and disadvantages of each one? (20 marks)

d. A cubic spine interpolates two points with a cubic polynomial. A cubic polynomial has four unknowns. Two points provide just two simultaneous equations. How do we constrain the solution to be a unique polynomial? (20 marks)

(Total: 100 marks)

Q 5

a. Consider the samples in Table 1.

i Fit an order 3 polynomial to these points. You can assume the MATLAB mldivide commands (or equivalently the \ operator) is available for the final step of the calculation, or state the answer in terms of the Moore-Penrose pseudo-inverse. As this final step is excessively long to evaluate in a paper-based exam, you are provided with the correct polynomial as follows to allow you to answer the remaining parts of this question:

$$p(t) = 0t^3 - 0.3973t^2 + 1.2483t - 0.0099$$

ii Differentiate the polynomial and hence estimate the derivative at $t = 1$.

iii Integrate the polynomial and hence estimate the integral $\int_0^{\pi} f(t)dt$.

iv Numerically integrate the samples using any of the numerical integration techniques that you have learned.

(80 marks)

Table 1: Samples for Q5.

t	0	$\frac{\pi}{5}$	$\frac{2\pi}{5}$	$\frac{3\pi}{5}$	$\frac{4\pi}{5}$	π
$f(t)$	0	0.5878	0.9511	0.9511	0.5878	0

b. These are in fact samples of the signal $f(t) = \sin(t)$. Using this fact, determine the error in the estimate of the derivative and the estimates of the integral above. (20 marks)

(Total: 100 marks)

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