



北京工业大学北京 – 都柏林国际学院  
BEIJING-DUBLIN INTERNATIONAL COLLEGE AT BJUT

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**SEMESTER 2 RESIT EXAMINATIONS**  
**ACADEMIC YEAR 2021/2022**

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**EEEN3018J Numerical Methods and Optimization**

Prof. Thomás Ward  
Prof. John T. Sheridan  
Dr John Healy  
Dr Declan Delaney\*

**BJUT Student ID:** \_\_\_\_\_ **UCD Student ID:** \_\_\_\_\_

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

**Honesty Pledge:** \_\_\_\_\_ **(Signature)**

**Time Allowed: 2 HOURS**

**Instructions for Candidates**

This examination paper contains **FOUR** questions.  
Answer **EITHER** question 1 or question 2, **AND EITHER** question 3 or question 4 for a maximum of 200 marks.

All four questions carry equal marks.

The distribution of marks for the parts of each question is indicated in ( ) brackets

**Instructions for Invigilators**

Non-programmable calculators are permitted.  
No rough-work paper is to be provided for candidates.

## Formulas

These formulas are provided without further context.

$$f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{(x_n - x_{n-1})f(x_n)}{f(x_n) - f(x_{n-1})}$$

$$\begin{pmatrix} \sum_{i=1}^n x_i^2 & \sum_{i=1}^n x_i \\ \sum_{i=1}^n x_i & n \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^n x_i y_i \\ \sum_{i=1}^n y_i \end{pmatrix}$$

$$A^T A c = A^T y$$

$$a_i = \frac{y_{i+1} - y_i}{x_{i+1} - x_i}$$

$$b_i = y_i$$

$$\begin{bmatrix} \frac{2h_0}{3} & \frac{h_0}{3} & 0 & \dots & 0 & 0 & 0 \\ \frac{h_0}{3} & \frac{2}{3}(h_1 + h_0) & \frac{h_1}{3} & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & \frac{h_{n-2}}{3} & \frac{2}{3}(h_{n-2} + h_{n-1}) & \frac{h_n}{3} \\ 0 & 0 & 0 & \dots & 0 & \frac{h_{n-1}}{3} & \frac{2h_{n-1}}{3} \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_{n-1} \\ b_n \end{bmatrix} = \begin{bmatrix} \frac{f_0}{h_0} - \alpha \\ \frac{f_1}{h_1} - \frac{f_0}{h_0} \\ \vdots \\ \frac{f_{n-1}}{h_{n-1}} - \frac{f_{n-2}}{h_{n-2}} \\ \beta - \frac{f_{n-1}}{h_{n-1}} \end{bmatrix}$$

$$hf\left(\frac{h}{2}\right)$$

$$h[f(0) + f(h)]$$

$$\frac{h}{3}[f(0) + 4f(h) + f(2h)]$$

$$x_{n+1} = x(t_n) + \Delta t f(t_n, x_n)$$

$$x_{n+1} = x_n + \frac{1}{2}\Delta t [f(t_n, x_n) + f(t_n + \Delta t, x_{n+1}^p)]$$

$$k_1 = \Delta t f(t_n, x_n)$$

$$k_2 = \Delta t f(t_n + \alpha \Delta t, x_n + \beta k_1)$$

$$x_{n+1} = x_n + ak_1 + bk_2$$

**Q 1**

- a. Define the order of convergence of a root finding algorithm. (20 marks)
- b. Derive the formula for Newton's method. (30 marks)
- c. Using the function  $y = x^2 - 8$ , find the positive root using three iterations of Newton's Method with a starting point of  $x = 3$ . Either leave the answers to each step in fraction form or round to 7 or 8 decimal places. Compare to the correct answer of  $2\sqrt{2}$ . (30 marks)
- d. The order of convergence of Newton's method is 2, whereas that of the secant method is around 1.618. What is the significance of these numbers? In what other significant way do these two methods differ? (20 marks)

**(Total: 100 marks)****Q 2**

- a. State the definitions of the following numerical integration algorithms.
- i The midpoint rule.
  - ii The trapezoid rule.
  - iii Simpson's rule.
- (24 marks)
- b. The signal  $f(x) = 2 + \cos\left(\frac{\pi t}{2}\right)$  is sampled on the range  $t = 0$  s to  $t = 4$  s at regular intervals of 1 s, yielding the samples  $[3 \ 2 \ 1 \ 2 \ 3]$ , as shown in Fig. 1. Use Simpson's rule to estimate the d.c. term of the Fourier transform of the continuous signal. (The d.c. term is obtained by simply integrating the signal, as the Fourier kernel is  $e^0 = 1$ .) (30 marks)
- c. Is the result as expected? Justify your answer. (16 marks)

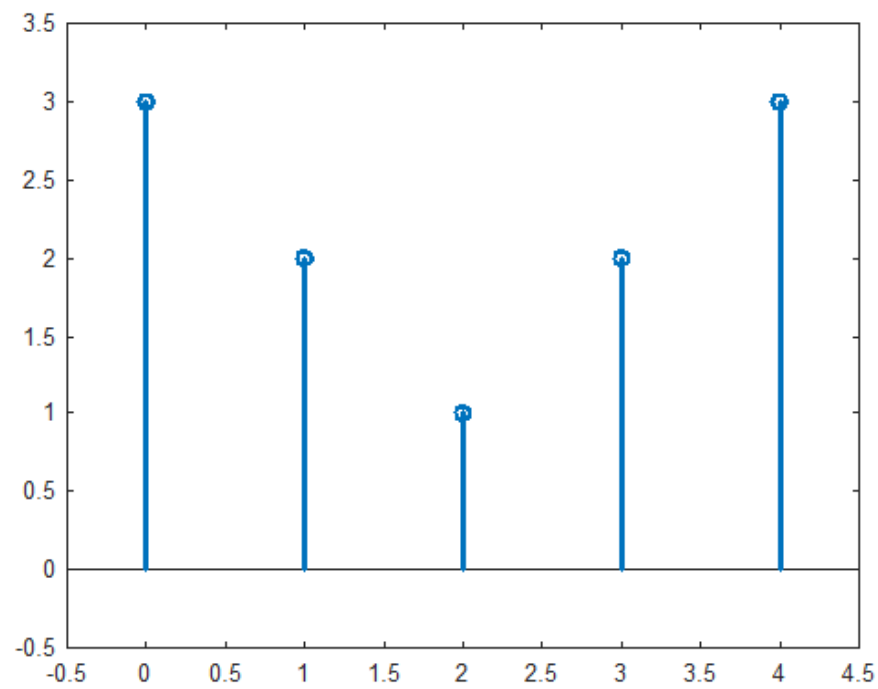


Figure 1: Samples for question (b).

- d.** Given the length 4 Discrete Fourier Transform (DFT) matrix below, find its LU decomposition.

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{pmatrix}$$

(30 marks)

**(Total: 100 marks)**

**Q 3**

a. From the following optimisation problem:

$$\begin{aligned} \max \quad & z(x) = (2, -3, 4, -1)x \\ \text{s.t.} \quad & \begin{pmatrix} 4 & 2 & -4 & 1 \\ 6 & 3 & 2 & 3 \end{pmatrix} x = \begin{pmatrix} 2 \\ 6 \end{pmatrix} \\ & x \geq 0 \end{aligned}$$

Write the optimisation problem in canonical form for the Basis B=(1,3)?

(30 marks)

b. From the following optimisation problem ( $\mathcal{P}$ ):

$$\begin{aligned} \max \quad & z(x) = (1, 3, 2)x \\ \text{s.t.} \quad & \begin{pmatrix} 1 & -2 & 1 \\ -1 & 3 & 2 \end{pmatrix} x = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \\ & x \geq 0 \end{aligned}$$

1. Determine the auxiliary problem ( $\mathcal{A}$ ) and determine a feasible Basis for the problem ( $\mathcal{P}$ ).
2. Use the determined Basis and use the simplex algorithm to find the problem solution.

(70 marks)

**(Total: 100 marks)**

**Q 4**

a. Determine if the following matrix is positive definite.

$$\begin{pmatrix} 1 & 3 & 2 \\ -4 & 4 & 4 \\ 1 & -2 & 4 \end{pmatrix}$$

(30 marks)

**b.** For the following function:

$$f(x, y) = 2x^2 + 4y^2 + 2xy - 4x + 14y - 7$$

Find the point  $[x, y]'$  derived after 2 iterations using Gradient Decent - Steepest decent search from the point:

$$[x, y] = [4, 3]$$

What is the location of this point  $[x, y]'$ ?  
what is the function value at the point  $[x, y]'$ ?  
Give your answer up to 3 decimal places.

Note:

For each line search work out the minimum point analytically.

(70 marks)

**(Total: 100 marks)**