



# Beijing-Dublin International College



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**SEMESTER I FINAL EXAMINATION – 2020/2021**

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**School of Mathematics and Statistics**  
**BDIC1029J & BDIC1025J Maths 1 (Advanced Mathematics)**

HEAD OF SCHOOL: Wenying WU  
MODULE COORDINATOR: Yanru PING  
OTHER EXAMINERS: Yuehong FENG, Rong YANG

**Time Allowed: 90 minutes**

## **Instructions for Candidates**

Answer ALL questions. The marks that each question carry is written as shown.

**BJUT Student ID:** \_\_\_\_\_ **UCD Student ID:** \_\_\_\_\_

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

**Honesty Pledge:** \_\_\_\_\_ **(Signature)**

## **Instructions for Invigilators**

Non-programmable calculators are permitted. NO dictionaries are permitted.  
No rough-work paper is to be provided for candidates.

**NOTE:** Answer **ALL** questions.

Time allowed is **90** minutes.

The exam paper has **2** sections on **5** pages, with a full score of 100 marks.

You are required to use the provided **Examination Book** only for answers.

### Section A: Gap-filling Questions

This section is worth a total of **75** marks, with each question worth **5** marks.

1. Evaluate

$$\lim_{t \rightarrow 0} \frac{\ln(1+t) - t}{t^2} = \underline{\hspace{2cm}}.$$

2. Find the limit

$$\lim_{x \rightarrow \infty} \frac{\sin x}{x} = \underline{\hspace{2cm}}.$$

3. Find the limit

$$\lim_{x \rightarrow 0} \frac{\sqrt[5]{x \arctan x + 1} - 1}{x^2} = \underline{\hspace{2cm}}.$$

4. Let  $f(x)$  be the function

$$f(x) = \frac{1}{(x+1)(x+2)}.$$

Find the higher order derivative  $f^{(2020)}(x) = \underline{\hspace{2cm}}.$

5. Let  $f(x)$  be the function

$$f(x) = \ln \sin x.$$

Find the differential  $df(x) = \underline{\hspace{2cm}}.$

**Advanced Mathematics (Module 1)**

6. Find the limit

$$\lim_{n \rightarrow \infty} \sqrt[n]{2^n + 1} = \underline{\hspace{2cm}}.$$

7. Given

$$\lim_{x \rightarrow 0} \frac{\ln [1 + h(x)]}{\tan x} = 5,$$

find the limit

$$\lim_{x \rightarrow 0} \frac{h(x)}{x} = \underline{\hspace{2cm}}.$$

8. Given

$$\lim_{x \rightarrow \infty} \left( \frac{x^2}{1+x} + ax + b \right) = 0,$$

evaluate  $a = \underline{\hspace{2cm}}$ , and  $b = \underline{\hspace{2cm}}$ .

9. Given

$$\lim_{x \rightarrow 0} \frac{\tan x}{\sin ax} = 2,$$

evaluate  $a = \underline{\hspace{2cm}}$ .

10. Given

$$y = x \sin 2x,$$

find the higher order derivative  $y^{(2020)} = \underline{\hspace{2cm}}$ .

11. If the following function is continuous at  $x = 0$ ,

$$f(x) = \begin{cases} (1 + \sin x)^{\frac{2}{x}}, & \text{if } x \neq 0; \\ A, & \text{if } x = 0, \end{cases}$$

try to determine the value of  $A = \underline{\hspace{2cm}}$ .

**Advanced Mathematics (Module 1)**

**12.** Given

$$\begin{cases} x = te^t, \\ e^t + e^y = 2, \end{cases} \quad t \text{ being a parameter, } t \in \mathbb{R},$$

evaluate

$$\frac{dy}{dx} = \underline{\hspace{2cm}}.$$

**13.** For

$$y = \left( \frac{x}{x+1} \right)^x,$$

try to find  $\frac{dy}{dx} = \underline{\hspace{2cm}}.$

**14.** Evaluate

$$\lim_{x \rightarrow 1} \left[ \frac{1}{x-1} - \frac{3}{(x^3-1)} \right] = \underline{\hspace{2cm}}.$$

**15.** Given  $f(x) = x + \sin x$ ,  $x \in [0, 2\pi]$ , the interval(s) of concave-up for  $f(x)$  is/are \_\_\_\_\_,  
and the point(s) of inflection is/are \_\_\_\_\_.

**Advanced Mathematics (Module 1)**

**Section B: Extended Answer Questions**

This section is worth a total of **25** marks. The marks of every question is as shown.

- 16.** (8 marks) Suppose  $f(x)$  is a continuous function over the interval  $[0, 2]$ .  $f(x)$  is derivable on the point  $(0, 2)$ , with evaluation  $f(2) = 0$ .

Try to prove that there exists at least one number  $\xi \in (0, 2)$ , such that

$$f'(\xi) = -\frac{3f(\xi)}{\xi}.$$

- 17.** (10 marks) Given that a function

$$y = f(x) = ax^3 + bx^2 + cx + d$$

has an extreme value  $y = 0$  occurring at  $x = 0$ , and  $(1, 1)$  is an inflection point on the curve  $y = f(x)$ .

Try to determine the values of  $a$ ,  $b$ ,  $c$  and  $d$ .

- 18.** (7 marks) Prove

$$xe^x + ye^y \geq (x + y)e^{\frac{x+y}{2}}, \quad \forall x \geq y > -2.$$

**Advanced Mathematics (Module 1)**

USEFUL FORMULAE

$$\frac{d}{dx}C = 0$$

$$\frac{d}{dx}x^\mu = \mu x^{\mu-1}$$

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x$$

$$\frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}\sec x = \sec x \tan x$$

$$\frac{d}{dx}\csc x = -\csc x \cot x$$

$$\frac{d}{dx}a^x = a^x \ln a$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}\log_a x = \frac{1}{x \ln a}$$

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\arctan x = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\operatorname{arccot} x = -\frac{1}{1+x^2}$$

## Glossary

|                             |       |
|-----------------------------|-------|
| Concave-up/down             | 凹/凸   |
| Curve                       | 曲线    |
| Decreasing                  | 递减    |
| Derivable                   | 可导的   |
| Derivative                  | 导数    |
| Differentiable              | 可微分的  |
| Differential                | 微分    |
| Discontinuity               | 不连续   |
| Higher order derivative     | 高阶导数  |
| Horizontal asymptote        | 水平渐近线 |
| Infinitesimal               | 无穷小量  |
| Interval                    | 区间    |
| Local maximum               | 极大值   |
| Logarithmic differentiation | 对数求导法 |
| Minimum                     | 最小值   |
| Monotonic increasing        | 单调递增  |
| Point of inflection         | 拐点    |
| Tangent line                | 切线    |