



# Beijing-Dublin International College



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**SEMESTER I FINAL EXAMINATION – 2021/2022**

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**School of Mathematics and Statistics**  
**BDIC1029J & BDIC1040J Maths 1 (Advanced Mathematics)**

HEAD OF SCHOOL: Wenying WU  
MODULE COORDINATORS: Bin ZHENG, Yanru PING  
OTHER EXAMINERS: Rong YANG, Nicholas A. HOUSTON

**Time Allowed: 90 minutes**

## **Instructions for Candidates**

Answer ALL questions. The marks that each question carry is written as shown.

**BJUT Student ID:** \_\_\_\_\_ **UCD Student ID:** \_\_\_\_\_

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

**Honesty Pledge:** \_\_\_\_\_ **(Signature)**

## **Instructions for Invigilators**

Non-programmable calculators are permitted. NO dictionaries are permitted.  
No rough-work paper is to be provided for candidates.

**NOTE:** Answer **ALL** questions.

Time allowed is **90** minutes.

The exam paper has **2** sections on **4** pages, with a full score of 100 marks.

You are required to use only the provided **Examination Book** for answers.

## SECTION A — Brief Answer Questions

This section is worth a total of **80** marks, with each question worth **5** marks.

1. Evaluate the limit

$$\lim_{x \rightarrow 3} \frac{x^2 - 9}{x^2 - 2x - 3} = \underline{\hspace{2cm}}.$$

2. Evaluate the limit

$$\lim_{x \rightarrow +\infty} (\sqrt{x^2 + 2x} - x) = \underline{\hspace{2cm}}.$$

3. Evaluate the limit

$$\lim_{x \rightarrow 0} \frac{\sqrt{\cos x} - 1}{x} = \underline{\hspace{2cm}}.$$

4. Let  $f(x) = xe^x$ , compute the 20-th derivative of  $f$ .

$$f^{(20)}(x) = \underline{\hspace{2cm}}.$$

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5. Find the differential for the function  $f(x) = 3 \cos^2 x$ ,  $df =$  \_\_\_\_\_. Use the differential to approximate the change in  $f(x)$  when we have a small change in  $x$ ,  $dx = 0.01$ , at  $x = \pi/4$ , and estimate  $f(\pi/4 + 0.01) \approx$  \_\_\_\_\_.

6. Evaluate the limit

$$\lim_{x \rightarrow +\infty} x^{\frac{1}{1+\ln x}} \text{_____}.$$

7. If  $f(9) = 1$ ,  $f'(9) = 3$ ,  $g(1) = 4$ ,  $g'(1) = -2$ , and  $h(x) = x^{3/2}g(f(x))$ , then

$$h'(9) = \text{_____}.$$

8. Given  $f(x) = \frac{2x^2}{x-3x^2}$ , then the equation of the horizontal asymptote of the curve  $y = f(x)$  is  $y =$  \_\_\_\_\_, whereas the equation of the vertical asymptote is  $x =$  \_\_\_\_\_.

9. Evaluate the limit

$$\lim_{x \rightarrow 0} \frac{\tan^2(x + \frac{\pi}{4}) - \tan^2(\frac{\pi}{4})}{x} = \text{_____}.$$

10. Given  $y = e^{\frac{-x^2}{2}}$ , compute the first, second, and third derivatives of  $y$ .

$$y' = \text{_____}, \quad y'' = \text{_____}, \quad y''' = \text{_____}.$$

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11. Evaluate the limit

$$\lim_{t \rightarrow 0} \left( \frac{1}{t} - \frac{1}{e^t - 1} \right) = \underline{\hspace{2cm}}.$$

12. Given

$$\begin{cases} x &= 3t^2 + 1, \\ y &= t^3 - 2t^2, \end{cases} \quad \text{where } t \in \mathbb{R} \text{ is a parameter.}$$

Compute  $\frac{dy}{dx} = \underline{\hspace{2cm}}$ ,  $\frac{d^2y}{dx^2} = \underline{\hspace{2cm}}$ .

13. Given  $y = (\cos x)^{\sin x}$ , then  $\frac{dy}{dx} = \underline{\hspace{2cm}}$ .

14. Given  $y - e^{xy} = 0$ , then  $y' = \underline{\hspace{2cm}}$ .

15. Find the values of the constants  $a$  and  $b$ , such that the function

$$f(x) = \begin{cases} ax + b & x < 0 \\ x^3 + 2x + 1 & x \geq 0, \end{cases}$$

is differentiable at  $x = 0$ . (Hint: Find  $a = \underline{\hspace{2cm}}$ ,  $b = \underline{\hspace{2cm}}$ .)

16. Given  $f(x) = 2x^3 - 3x^2 - 12x + 1$ , defined on a closed interval  $x \in [-2, 0]$ . Its absolute maximum value is  $\underline{\hspace{2cm}}$ , absolute minimum value is  $\underline{\hspace{2cm}}$ .

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**SECTION B — Extended Answer Questions**

Write your answers on the **Examination Book** provided.

This section is worth a total of **20** marks, with each question worth 5 marks.

**17.** Suppose  $f(x)$  is continuous and differentiable for all real numbers. If  $-1 \leq f'(x) \leq 3$  and  $f(5) = 6$ , what is the largest  $f(x)$  can be at  $x = 1$ ?

**18.** Prove the identity:  $\arcsin x + \arccos x = \frac{\pi}{2}$ , where  $x \in [-1, 1]$ .

**19.** Evaluate the limit

$$\lim_{x \rightarrow 0} \frac{x - \sin x}{x^3}$$

by the L'Hospital's Rule.

**20.** Prove the inequality:  $1 + 2 \ln x \leq x^2$ , for  $x > 0$ .

## Glossary

|                      |       |
|----------------------|-------|
| Absolute maximum     | 最大值   |
| Absolute minimum     | 最小值   |
| Approximate          | 近似    |
| Continuous           | 连续    |
| Curve                | 曲线    |
| Derivative           | 导数    |
| Differentiable       | 可微分的  |
| Differential         | 微分    |
| Estimate             | 估计    |
| Horizontal asymptote | 水平渐近线 |
| Identity             | 恒等式   |
| Inequality           | 不等式   |
| Interval             | 区间    |
| Limit                | 极限    |
| Parameter            | 参数    |
| Vertical asymptote   | 铅垂渐近线 |