Beijing-Dublin International College

SEMESTER I FINAL EXAMINATION – 2021/2022

School of Mathematics and Statistics
BDIC1041J Maths 2 (Advanced Mathematics)

HEAD OF SCHOOL: Wenying WU
MODULE COORDINATOR: Yanru PING

Time Allowed: 90 minutes

Instructions for Candidates

Answer ALL questions. The marks that each question carry is written as shown.

BJUT Student ID: _____ **UCD Student ID: _____**

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge：_____ (Signature)

Instructions for Invigilators

Non-programmable calculators are permitted. NO dictionaries are permitted.
No rough-work paper is to be provided for candidates.

NOTE: Answer **ALL** questions.

Time allowed is **90** minutes.

The exam paper has **2** sections on **6** pages, with a full score of 100 marks.

You are required to use only the provided **Examination Book** for answers.

SECTION A — Brief Answer Questions

This section is worth a total of **70** marks, with each question worth **5** marks.

1. Compute

$$\int \frac{1}{\sqrt{3}\cos x - \sin x} dx = _____.$$

2. Compute

$$\int \cot^3 x dx = _____.$$

3. Compute

$$\int \frac{1}{x(2 + x^{2021})} dx = _____.$$

4. Compute

$$\int x \ln x dx = _____.$$

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5. Compute

$$\int \frac{x^3}{4+x^8}dx = _____.$$

6. Compute

$$\int \frac{\ln(x+\sqrt{x^2+1})}{\sqrt{x^2+1}}dx = _____.$$

7. Compute

$$\lim_{x\to 0^+} \frac{\int_0^{\sqrt{x}}(1-\cos t^2)dt}{x^{\frac{5}{2}}}$$

8. Compute

$$\int_{-1}^{1}\left\{\frac{\sin^3 x}{\sqrt{1+x^2}} - \frac{x^2}{1+\sqrt{1-x^2}}\right\}dx = _____.$$

9. Compute

$$\int_0^{\frac{\pi}{2}} \sqrt{1-\sin 2x}dx$$

10. Evaluate the definite integral

$$\int_0^1 x \cdot (1-x)^{50} dx = _____.$$

11. Determine if the following improper integral is convergent or not:

$$\int_0^{+\infty} e^{-3x} dx.$$

If this integral is convergent, find its value: _____.

12. Try to estimate the following limit by making use of the definition of definite integral:

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{k}{\sqrt{n^4 + n^2 k^2}}.$$

13. Express the complex number $\sqrt{3}+i$ as polar form $\sqrt{3}+i =$ _____ and polar exponential form $\sqrt{3}+i =$ _____, and then calculate $(\sqrt{3}+i)^{12} =$ _____.

14. Given $z = -1 + i$ is a root for a polynomial equation

$$z^4 + 6z^3 + 15z^2 + 18z + 10 = 0.$$

Find the other three roots of the equation.

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SECTION B — Extended Answer Questions

Write your answers on the **Examination Book** provided.

This section is worth a total of **30** marks. The marks of each question are as shown.

15. (**10 marks**) Suppose that $f(x)$ is continuous on $[0,1]$, firstly prove that

$$\int_0^\pi x \cdot f(\sin x)dx = \frac{\pi}{2}\int_0^\pi f(\sin x)dx.$$

Secondly , calculate

$$\int_0^\pi x \sin^3 x \cos^4 x dx$$

16. (**12 marks**) In the two-dimensional plane, there is a region,denoted by D, enclosed by a parabola $y = x^2$ and a parabola $x = y^2$.

(a) Find the area of this region.(4 marks)

(b) Find the volume of the solid obtained by revolving the region about the vertical line $x = -1$.(4 marks)

(c) Express the length of the circumference of the region by means of definite integral.(4 marks)

17. (**8 marks**) Suppose that $f(x)$ is continuous on $[0,1]$, then prove that

$$\int_0^1 e^{f(x)}dx \int_0^1 e^{-f(x)}dx \geq 1$$

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Useful formulae

$$\int x^{\alpha} dx = \frac{1}{\alpha+1} x^{\alpha+1} + c \ , (\alpha \neq -1)$$

$$\int \frac{1}{x} dx = \ln x + c$$

$$\int \frac{1}{\sqrt{x^2 \pm 1}} dx = \ln |(x + \sqrt{x^2 \pm 1})| + c$$

$$\int \frac{1}{\cos^2 x} dx = \tan x + c$$

$$\int \frac{1}{\sin^2 x} dx = -\cot x + c$$

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$$\int \sin x dx = -\cos x + c$$

$$\int \cos x dx = \sin x + c$$

$$\int \tan x dx = -\ln|\cos x| + c$$

$$\int \cot x dx = \ln|\sin x| + c$$

$$\int \sec x dx = \ln|\sec x + \tan x| + c$$

$$\int \csc x dx = -\ln|\csc x + \cot x| + c$$

$$\int a^x dx = \frac{1}{\ln a} a^x + c$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \arctan \frac{x}{a} + c$$

$$\int \frac{1}{\sqrt{1 - x^2}} dx = \arcsin x + c$$

Glossary

Antiderivative	原函数
Boundary	边界
Circumference	周长
Coefficient	系数
Polar exponential form	复数的指数表达
Curve	曲线
Definite integral	定积分
Even function	偶函数
Polar form	复数的极坐标表达
Convergent	收敛的
Improper integral	广义积分
Odd function	奇函数
Show that	证明
Region	区域
definition	定义
Volume	体积
Area	面积