



Beijing-Dublin International College



AUTUMN TRIMESTER EXAMINATION - 2020/2021

EEEN3003J: Signals and Systems

MODULE COORDINATOR: Dr. Barry Cardiff

Time Allowed: 1 Hours

Instructions for Candidates

Answer any **two** of the 3 questions. All questions carry equal marks.

The percentages in the right margin give an approximate indication of the relative importance of each part of the particular question.

All rough work must be entered in your answer books.

BJUT Student ID: _____ **UCD Student ID:** _____

I have read and clearly understand the examination rules of both Beijing University of Technology and University College Dublin. I am aware of the punishment for violating the rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge: _____ **(Signature)**

Instructions for Invigilators

Non-programmable calculators are permitted.

No rough-work paper is to be provided to candidates

Question: 1

1.1 Consider $h(t)$ as shown in Figure 1:

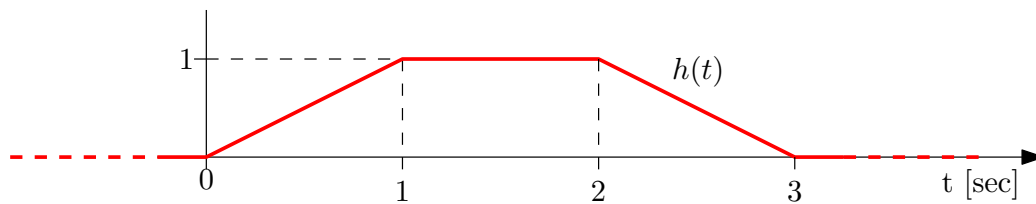


Figure 1: Pulse $h(t)$ to be considered in Question 1.1.

Using the Laplace transform evaluate $y(t) = (u \star h)(t)$, the convolution of $h(t)$ and $u(t)$, where $u(t)$ is the unit step function. Your answer should be expressed as a piecewise function having a format similar to:

$$y(t) = \begin{cases} \text{some function} & \text{for } t < a \\ \text{another function} & \text{for } a \leq t < b \\ \dots \text{etc} & \dots \text{etc} \end{cases}$$

Note: You must use the Laplace transform to compute $Y(s)$ and then use the inverse Laplace transform to compute $y(t)$; all steps must be shown, and if you use any 'well known' Laplace transform identities / properties, you must state them. [40%]

1.2 Sketch $y(t)$ (a clear sketch is required with correctly labelled axis and units). [10%]

1.3 Let $x(t)$ be the input of a system having impulse response $g(t)$, where $x(t)$ and $g(t)$ are defined below.

$$g(t) \triangleq \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2} \quad \text{and} \quad x(t) = \sin(\omega t) \quad \forall t$$

Compute the system's time domain output.

Note: You may find the following useful: $\int_{-\infty}^{\infty} e^{-u^2} du = \sqrt{\pi}$. [40%]

1.4 Is this system a low-pass, high-pass, band-pass or notch filter? Explain your answer (using equations). [10%]

Question: 2

2.1 Prove, showing your steps, the 'final value theorem', i.e. that:

$$\lim_{s \rightarrow 0} sF(s) = \lim_{t \rightarrow \infty} f(t)$$

where $F(s)$ is the Laplace transform of $f(t)$. [20%]

2.2 Does this final value theorem work for the following function? Prove your answer.

$$f(t) = \begin{cases} t & 0 \leq t < 1 \\ 0 & \text{elsewhere} \end{cases}$$

[10%]

2.3 Find the partial fraction expansion of:

$$F(s) = \frac{1}{s(s-1)(s+1)^2}$$

Don't just give the answer - show your proof. [30%]

2.4 Hence or otherwise compute $f(t) = \mathcal{L}^{-1}\{F(s)\}$. [20%]

2.5 Consider the following ODE:

$$\frac{d^3}{dt^3}g(t) + 3\frac{d^2}{dt^2}g(t) + 4\frac{d}{dt}g(t) + 2g(t) = f(t)$$

What is its transfer function ($\triangleq \frac{G(s)}{F(s)}$)?

Hint: the denominator should have at least one root at $s = -1$; this should help to simplify the answer. [20%]

Question: 3

If $F(j\omega)$ and $G(j\omega)$ is the Fourier Transform of $f(t)$ and $g(t)$ respectively, i.e.:

$$f(t) \xleftrightarrow{\mathcal{F}} F(j\omega) \quad \text{and} \quad g(t) \xleftrightarrow{\mathcal{F}} G(j\omega)$$

prove the following properties:

3.1 $\alpha f(t) + \beta g(t) \xleftrightarrow{\mathcal{F}} \alpha F(j\omega) + \beta G(j\omega)$ for any set of constants α and β [20%]

3.2 $f(t - \tau) \xleftrightarrow{\mathcal{F}} e^{-j\tau\omega} F(j\omega)$ [10%]

3.3 $f(\lambda t) \xleftrightarrow{\mathcal{F}} \frac{1}{|\lambda|} F(j\frac{\omega}{\lambda})$ for any constant λ . Hint: be careful with the sign when doing variable substitution (λ can be positive or negative) [10%]

3.4 If $f(t)$ is real, then $F(j\omega) = F^*(-j\omega)$ [10%]

3.5 Consider the signal:

$$f(t) = A \sin(\omega_o t + \phi)$$

Derive, from first principles, the Fourier Transform of $f(t)$. You will get 0% for just quoting the answer as it is easily available in the 'useful formula' at the end of this paper! [25%]

3.6 Consider a system having Transfer Function (in the Fourier domain) of:

$$H(j\omega) = \frac{1}{1 + j\omega}$$

Compute its output when the input is $A \sin(\omega_o t + \phi)$. [25%]

Fourier Series

$$c_k = \frac{1}{T} \int_T f(t) e^{-jk\omega_o t} dt \quad , \quad \omega_o = \frac{2\pi}{T}$$

$$f(t) = \mu + \sum_{k=1}^{\infty} a_k \cos(k\omega_o t) + b_k \sin(k\omega_o t)$$

$$a_k = 2\Re\{c_k\} \quad , \quad b_k = -2\Im\{c_k\}$$

Fourier Transform

$$F(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(j\omega) e^{j\omega t} d\omega$$

Laplace Transform Pairs

$$f(t) \xleftrightarrow{\mathcal{L}} F(s)$$

$$\sin(at) \xleftrightarrow{\mathcal{L}} \frac{a}{s^2 + a^2}$$

$$\cos(at) \xleftrightarrow{\mathcal{L}} \frac{s}{s^2 + a^2}$$

$$t^n \xleftrightarrow{\mathcal{L}} \frac{n!}{s^{n+1}}$$

$$e^{at} \xleftrightarrow{\mathcal{L}} \frac{1}{s - a}$$

$$\delta(t) \xleftrightarrow{\mathcal{L}} 1$$

$$1 \xleftrightarrow{\mathcal{L}} \frac{1}{s}$$

Trigonometric Formula

$$\cos(A \pm B) = \cos(A) \cos(B) \mp \sin(A) \sin(B)$$

$$\sin(A \pm B) = \sin(A) \cos(B) \pm \cos(A) \sin(B)$$

$$e^{j\theta} = \cos(\theta) + j \sin(\theta)$$

Fourier Transform Pairs

$$e^{j\omega_o t} \xleftrightarrow{\mathcal{F}} 2\pi\delta(\omega - \omega_o)$$

$$u(t) \xleftrightarrow{\mathcal{F}} \pi\delta(\omega) + \frac{1}{j\omega}$$

Fourier Properties

$$f(\lambda t) \xleftrightarrow{\mathcal{F}} \frac{1}{|\lambda|} f\left(j\frac{\omega}{a}\right)$$

$$f(t - \tau) \xleftrightarrow{\mathcal{F}} e^{-j\omega\tau} F(j\omega)$$

$$\frac{d}{dt} f(t) \xleftrightarrow{\mathcal{F}} j\omega F(j\omega)$$

Laplace Properties

$$f(at) \xleftrightarrow{\mathcal{L}} \frac{1}{a} f\left(j\frac{s}{a}\right)$$

$$e^{-at} f(t) \xleftrightarrow{\mathcal{L}} F(s + a)$$

$$\int_0^t f(\tau) d\tau \xleftrightarrow{\mathcal{L}} \frac{1}{s} F(s)$$

$$\frac{d}{dt} f(t) \xleftrightarrow{\mathcal{L}} s f(s) - f(0)$$

$$\frac{d^n}{dt^n} f(t) \xleftrightarrow{\mathcal{L}} s^n F(s) - \sum_{k=1}^n s^{n-k} \frac{d^{k-1}}{dt^{k-1}} f(t) \Big|_{t=0^-}$$

$$(-t) f(t) \xleftrightarrow{\mathcal{L}} \frac{d}{ds} F(s)$$

Integrals

$$\int \cos(x) = \sin(x)$$

$$\int \sin(x) = -\cos(x)$$

$$\int u dv = uv - \int v du$$

Partial fractions

$$\text{If } F(s) = \frac{N(s)}{\prod_i (s - p_i)^{k_i}} = \sum_i \left\{ \frac{r_{i,1}}{s - p_i} + \frac{r_{i,2}}{(s - p_i)^2} + \dots + \frac{r_{i,k_i}}{(s - p_i)^{k_i}} \right\}$$

$$\text{then } r_{i,j} = \frac{1}{(k_i - j)!} \lim_{s \rightarrow p_i} \frac{d^{k_i-j}}{ds^{k_i-j}} (s - p_i)^{k_i} F(s)$$

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