Beijing-Dublin International College

SEMESTER II FINAL EXAMINATION – 2021/2022

Beijing-Dublin International College

BDIC1042J Maths 3 (Advanced Mathematics; Finance)

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MODULE LECTURER: Xin Liu

Time Allowed: 90 minutes

Instructions for Candidates

Your answers to ALL the questions should be written on the Answer Book.
The marks that each question carry are as shown.

BJUT Student ID: _____ **UCD Student ID:** _____

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge： _____ **(Signature)**

Instructions for Invigilators

No calculators are permitted. NO dictionaries are permitted.
No rough-work paper is to be provided for candidates.

SECTION A — MULTIPLE CHOICE QUESTIONS

In each question, choose **at most one** option.

Write your answers on the **Examination Book** provided.

This section is worth a total of **15** marks, with each question worth **3** marks.

1. How to obtain the shape of the following paraboloid in space?

$$z = 4x^2 + y^2$$

(a) Step 1: Starting from a planar curve $z = x^2$.

Step 2: Rotating about the z-axis to obtain: $z = x^2 + y^2$.

Step 3: Doing compression in the x-direction by $x \to 2x$, to obtain: $z = (2x)^2 + y^2$.

(b) Step 1: Starting from a planar curve $z = x^2$.

Step 2: Rotating about the x-axis to obtain: $z = x^2 + y^2$.

Step 3: Doing compression in the x-direction by $x \to 2x$, to obtain: $z = (2x)^2 + y^2$.

(c) Step 1: Starting from $z = 4x^2 - y^2$.

Step 2: Doing reflection by $y \to -y$, to obtain: $z = 4x^2 + y^2$.

(d) Step 1: Starting from a plane $z = 2x + y$.

Step 2: Rotating about the line $x + y + z = 0$ to obtain: $z = (2x)^2 + y^2$.

2. Which of the following statements is correct?

(a) In the three-dimensional space, there are only 3 paths to approach a point (x_0, y_0, z_0);

(b) In the two-dimensional xy-plane, there are only 2 paths to approach a point (x_0, y_0);

(c) On the one-dimensional x-axis, there are only 2 paths to approach a point x_0;

(d) On the one-dimensional x-axis, there is only 1 path to approach a point x_0.

3. The following quadratic surface is a _____:

$$x^2 + 2z^2 - 6x - y + 10 = 0.$$

(a) sphere; (b) ellipsoid; (c) hyperboloid; (d) paraboloid.

4. Consider a function

$$f(x,y) = \begin{cases} \frac{3x^2y}{x^2+y^2}, & \text{if } (x,y) \neq (0,0), \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$$

(a) It is a continuous function everywhere.

(b) It is discontinuous at the point $(0,0)$.

(c) It is discontinuous along the line $x = 0$.

(d) It is discontinuous along the line $y = 0$.

5. The total differential of a two-variable function $z = \left(x^2 - y^2\right) e^y$ is given by

(a) $dx + dy = \left(x^2 - y^2\right) e^y dz$;

(b) $dz = e^y \left[2xdx + \left(x^2 - y^2 - 2y\right) dy\right]$;

(c) $dz = \left(x^2 - y^2 + 2x - 2y\right) \left[e^y dx + \left(x^2 - y^2\right) dy\right]$;

(d) $dz = 2xe^y dx + \left(x^2 - y^2 - 2y\right) dy$.

BDIC1042J Advanced Mathematics (Module 3; Finance)

SECTION B — GAP-FILLING QUESTIONS

Write your answers on the **Examination Book** provided.
This section is worth a total of **40** marks, with each question worth **4** marks.

6. Consider the plane $\Sigma:\ 3x-4y+z=10$ and the points $A\,(2,3,-1)$, $B\,(1,2,2)$. Find the equation of another plane Σ' which passes through A and B and is perpendicular to Σ: __

7. Let C be a curve in space

$$C:\begin{cases} x+y^2-z^2 &= 0,\\ x+y-2z &= 0. \end{cases}$$

The equation of the projection of C onto the xz-plane is _____.

8. Let C be a horizontal curve given by $\begin{cases} \cos(x-y) &= x+y,\\ z &= 3. \end{cases}$ Suppose Σ is a cone formed by moving a line L along C — that is, regarding C as a trajectory — while keeping L passing through the origin point $(0,0,0)$. The equation of the cone Σ is _____.

9. Evaluate the limit:

$$\lim_{(x,y)\to(0,0)}\frac{\sqrt{3}xy}{x+\sqrt{3}y}=\text{_____}.$$

10. Evaluate the limit:

$$\lim_{(x,y)\to(0,0)}x^2y^2\ln\left(x^2+y^2\right)=\text{_____}.$$

11. On a hot day, extreme humidity makes us think the temperature is higher than it really is. Hence a quantity, *Heat Index*, denoted as I, is devised to describe the combined effects of temperature and humidity, i.e., $I = f(T, H)$. The following table gives some data of I depending on T and H.

Heat Index (I) as a function of temperature (T) and humidity (H)

(T = Temperature, unit as °C; H = Humidity, unit as percent)

T \ H	50	55	60	65	70	75	80	85	90
32	96	98	100	103	106	109	112	115	119
33	100	103	105	108	112	115	119	123	128
34	104	107	111	114	118	122	127	132	137
35	109	113	116	121	125	129	135	141	146
36	114	118	123	127	132	138	144	150	157
37	119	124	129	135	141	147	154	161	168

At the point $T = 35°\text{C}$ and $H = 70$ percent, use the data from the table to approximate the following partial derivatives:

$$\frac{\partial I}{\partial T} \approx \frac{\Delta I}{\Delta T} = \underline{\qquad\qquad}, \qquad \frac{\partial I}{\partial H} \approx \frac{\Delta I}{\Delta H} = \underline{\qquad\qquad},$$

where ΔT and ΔH are the minimum variance of T and H, respectively, near the studied point.

12. Suppose that the equation $F(x, y, z) = 0$ implicitly defines each of the three variables x, y, and z as functions of the other two: $z = z(x, y)$, $y = y(x, z)$ and $x = x(y, z)$. If F is differentiable and F_x, F_y, and F_z are all nonzero, compute

$$\frac{\partial z}{\partial x}\frac{\partial x}{\partial y}\frac{\partial y}{\partial z} = \underline{\qquad\qquad}\ \underline{\qquad\qquad}.$$

13. Consider a function $z = f(x, y) = x^2 + 3xy - y$. If x changes from 2 to 2.05 and y changes from 3 to 2.96, try to find an approximate value for $z(2.05, 2.96) =$ _____.

14. Let $f = axy$, where a is a constant. y is a function depending on x through the relation

$$x^2 + a^2 = y^2.$$

Evaluate $\frac{df}{dx} =$ _____.

15. Consider a function $z = f(x, y)$ and the following three vectors in the xy-plane,

$$\boldsymbol{u}_1 = \boldsymbol{i} + 2\boldsymbol{j}, \qquad \boldsymbol{u}_2 = 2\boldsymbol{i} + 2\boldsymbol{j}, \qquad \boldsymbol{u}_3 = 3\boldsymbol{i} - 4\boldsymbol{j}.$$

If the directional derivatives in $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$ are $D_{\hat{\boldsymbol{u}}_1} f = -\frac{1}{\sqrt{5}}$ and $D_{\hat{\boldsymbol{u}}_2} f = 0$, respectively, try to compute the directional derivative along $\boldsymbol{u}_3$: $D_{\hat{\boldsymbol{u}}_3} f =$ _____.

BDIC1042J Advanced Mathematics (Module 3; Finance)

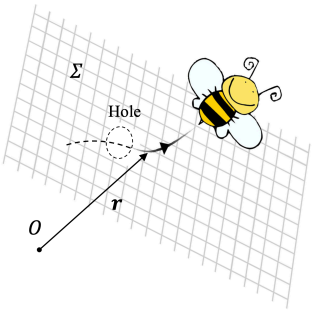
SECTION C — EXTENDED ANSWER QUESTIONS

Write your answers on the **Examination Book** provided.
This section is worth a total of **45** marks. The marks of each question are as shown.

16. (9 marks)

In the 3-dimensional space there is a flyscreen lying in a plane Σ,

$$\Sigma: \quad 2x + y - 2z = 1.$$



Let $\boldsymbol{r}$ denote the trajectory of a flying bee, as a function of time t:

$$\boldsymbol{r} = t\boldsymbol{i} + t^2\boldsymbol{j} + t^3\boldsymbol{k}, \qquad t \geq 0.$$

(a) Find all the time t when the bee is flying parallel to Σ.

(b) Find all the time t when the bee is flying perpendicular to Σ.

(c) There are holes in the screen Σ through which the bee passes.
Find the coordinates of the holes.
(Hint: *For Item (c), you may need to do polynomial factorization, in which* $(t+1)$ *is a factor.*)

17. (7 marks) Consider a hill in a shape described by the function

$$z = f(x, y) = x^2y^3 - 4y.$$

Let $P(2, -1, 0)$ be a point on the hill.

(a) Find the fastest increasing direction at P, denoted as $\hat{\boldsymbol{l}}_1$, and the directional derivative in that direction, denoted as $D_{\hat{\boldsymbol{l}}_1} f$. (3 *marks*)

(b) Find the fastest decreasing direction at P, denoted as $\hat{\boldsymbol{l}}_2$, and the directional derivative in that direction, denoted as $D_{\hat{\boldsymbol{l}}_2} f$. (1 *marks*)

(c) Find the direction $\hat{\boldsymbol{l}}_3$ at P along which the directional derivative has zero change. (3 *marks*)

18. (10 marks) Consider a function $z = z(x, y)$, where the relationship of x, y, z is defined by

$$F(u_1, u_2) \equiv F\left(\frac{y}{x}, \frac{z}{x}\right) = 0, \qquad u_1 = \frac{y}{x}, \quad u_2 = \frac{z}{x}.$$

Here F is a differentiable function, and $\frac{\partial F}{\partial u_2} \neq 0$. Try to compute

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \underline{\quad ? \quad}.$$

19. (8 marks) Classify every critical point of a function

$$z = f(x, y) = x^4 + y^4 - 4xy + 1, \qquad x, y \in \mathbb{R}$$

to be a local maximum/minimum point or a saddle point.

20. (11 marks) Let $f(x, y) = x^2 + 2y^2$, where x and y satisfy the constraint $x^2 + y^2 = 1$.

Use two methods to find all the extremum points of $f(x, y)$, and give the values of f at every extremum point. One of the methods must be the **Lagrangian multiplier method**.

Glossary

Approach	逼近
Approximate	近似
Axis	轴
Bee	蜜蜂
Combined effect	综合效应
Compression	压缩
Cone	锥面
Constraint	约束
Coordinate	坐标
Critical point	临界点
Decreasing	减少
Differentiable	可微
Directional derivative	方向导数
Discontinuous	不连续的
Ellipsoid	椭球面
Extremum	极值（点）
Factor	因子
Flyscreen	防虫网
Heat index	热指标
Hill	小山
Horizontal	水平的
Humidity	湿度
Hyperboloid	双曲面
Implicit	隐的
Increasing	增加
Lagrangian multiplier method	拉格朗日乘子法
Local	局域
Maximum	极大
Minimum	极小
Paraboloid	抛物面

Parallel	平行
Partial derivative	偏导数
Percent	百分之一
Perpendicular	垂直的
Planar	平面的
Polynomial factorization	多项式的因式分解
Projection	投影
Quadratic surface	二次曲面
Reflection	反射
Saddle point	鞍点
Temperature	温度
Total differential	全微分
Trajectory	轨迹
Variance	变化量