



Beijing-Dublin International College



SEMESTER II FINAL EXAMINATION – 2024/2025

Beijing-Dublin International College
BDIC1043J Maths 4 (Advanced Mathematics; Finance)

PRINCIPAL OF COLLEGE: Wenying WU
MODULE LECTURER: Xin LIU

Time Allowed: 95 minutes

Instructions for Candidates

Your answers to ALL the questions should be written on the Answer Book.

BJUT Student ID: _____ **UCD Student ID:** _____

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge: _____ **(Signature)**

Instructions for Invigilators

No calculators are permitted. NO dictionaries are permitted.
No rough-work paper is to be provided for candidates.

NOTE: The exam paper has **3** sections on **9** pages, with a full score of 100 marks.

SECTION A — MULTIPLE CHOICE QUESTIONS

In each question, choose **at most one** option.

Write your answers on the **Answer Book** provided.

This section is worth a total of **18** marks, with each question worth **3** marks.

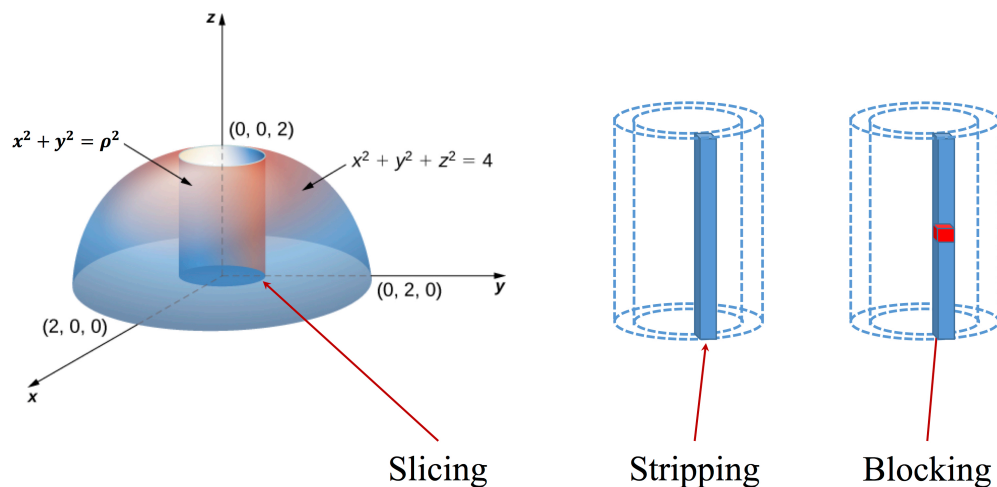
1. Consider a triple integral given in the rectangular coordinates (x, y, z) ,

$$I = \iiint_{\Omega} f(x, y, z) \, dx \, dy \, dz,$$

where the integration Ω is the upper half-ball,

$$\Omega = \{(x, y, z) \mid x^2 + y^2 + z^2 \leq 4, \quad z \geq 0\}.$$

Now let us change the coordinates to the cylindrical coordinate system, (ρ, θ, z) . If the following illustration demonstrates a type of integration strategy,



then determine which of the items below describes integration strategy?

- (a) $\int dz \int \rho d\rho \int d\theta f$; (b) $\int d\theta \int dz \int \rho d\rho f$; (c) $\int \rho d\rho \int d\theta \int dz f$; (d) $\int \rho d\rho \int dz \int d\theta f$.

BDIC1043J Advanced Mathematics (Module 4; Finance)

2. A triple integral over a region Ω is defined as

$$I = \iiint_{\Omega} f(x, y, z) dv = \lim_{v_i \rightarrow 0} \sum_{i=1}^n f(x_i, y_i, z_i) \Delta v_i,$$

where $\{\Delta v_1, \dots, \Delta v_i, \dots, \Delta v_n\}$ is a partition of Ω , with $i = 1, \dots, n$, and $(x_i, y_i, z_i) \in \Delta v_i$.

Which of the following statements is/are CORRECT?

- (a) The partition of Ω , $\{\Delta v_1, \dots, \Delta v_i, \dots, \Delta v_n\}$, is arbitrary. A necessary condition for the integral I to exist is the limitation behavior: $\Delta v_i \rightarrow 0, \forall i = 1, \dots, n$.
- (b) For this definition of integration there exists a requirement that all the small regions in the partition must be equal to each other, and equal to a standard element Δv ; i.e., $\Delta v_1 = \dots = \Delta v_i = \dots = \Delta v_n = \Delta v$, and the standard Δv goes to 0 in the limitation behavior.
- (c) The integral I is well defined (i.e., having a finite value), even if the integrand $f(x, y, z)$ is unbounded at finite points within the region Ω .
- (d) The integral I must be well defined (i.e., having a finite value), even if the region Ω contains infinitely many singular points.

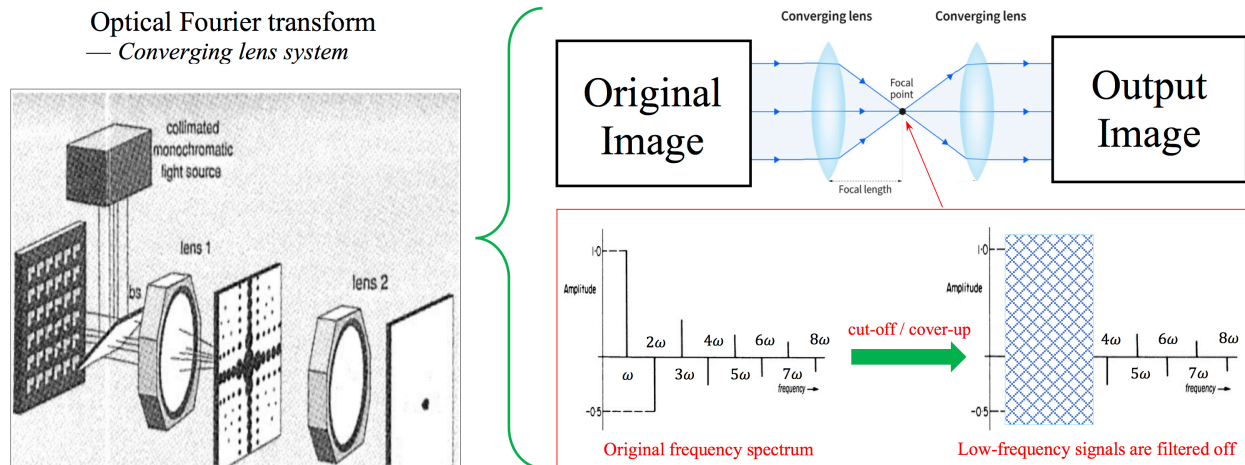
3. For the Dirac δ -function below, compute the integral

$$\int_{-\infty}^{\infty} \sin(\omega t) \delta(\omega - 2) d\omega = \text{_____}, \quad \forall t \in \mathbb{R}.$$

- (a) $2 \cos t \sin t$;
- (b) $\frac{e^{it} - e^{-it}}{2i}$;
- (c) $\frac{e^{i2t} - e^{-i2t}}{2}$;
- (d) $\frac{e^{i2t} + e^{-i2t}}{2}$.

BDIC1043J Advanced Mathematics (Module 4; Finance)

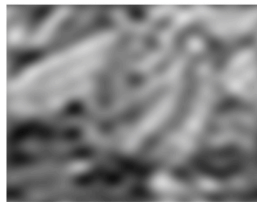
4. (*Optical Fourier Transform*) Let us consider the following converging lens system, where some particular frequency signals are filtered off from the frequency spectrum as shown.



Question:

If Original Image = , then Output Image = ?

Your options:



(a)



(b)



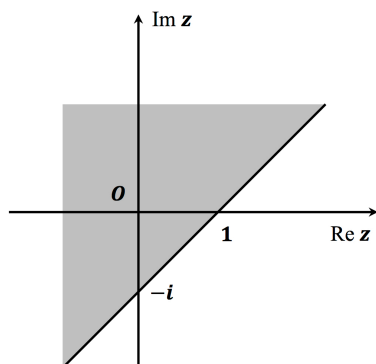
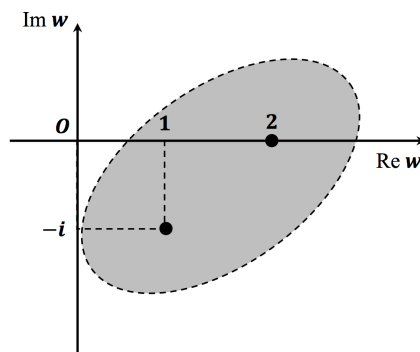
(c)



(d)

BDIC1043J Advanced Mathematics (Module 4; Finance)

5. Consider two complex numbers $z, w \in \mathbb{C}$ which are described by the following illustrations.

Complex number z Complex number w

Which item of mathematical expressions below for z and w is CORRECT?

- (a) $\left\{ z \in \mathbb{C} \mid \operatorname{Im} z \geq \operatorname{Re} z + 1 \right\}$ and $\left\{ w \in \mathbb{C} \mid |w - 2| + |w - 1 + i| < 4 \right\}.$
- (b) $\left\{ z \in \mathbb{C} \mid \operatorname{Im} z \leq \operatorname{Re} z + 1 \right\}$ and $\left\{ w \in \mathbb{C} \mid |w - 2| - |w - 1 + i| < 4 \right\}.$
- (c) $\left\{ z \in \mathbb{C} \mid |z - 2| + |z - 1 + i| \leq 4 \right\}$ and $\left\{ w \in \mathbb{C} \mid \operatorname{Re} w < \operatorname{Im} w + 1 \right\}.$
- (d) $\left\{ z \in \mathbb{C} \mid \operatorname{Re} z \leq \operatorname{Im} z + 1 \right\}$ and $\left\{ w \in \mathbb{C} \mid |w - 2| + |w - 1 + i| < 4 \right\}.$

6. The solution to the following equation is _____,

$$e^z = 3 + 3i, \quad z \in \mathbb{C}.$$

- (a) $i3\sqrt{2} \left(\frac{\pi}{16} + k\frac{\pi}{4} \right),$ where $k \in \mathbb{Z}.$
- (b) $3\sqrt{2} + i \left(\frac{\pi}{16} + k\frac{\pi}{4} \right),$ where $k = 0, 1, 2, 3.$
- (c) $\ln 3\sqrt{2} + i \left(\frac{\pi}{16} + k\frac{\pi}{4} \right),$ where $k = 0, 1, 2, 3.$
- (d) $\ln 3\sqrt{2} + i \left(\frac{\pi}{4} + 2k\pi \right),$ where $k \in \mathbb{Z}.$

BDIC1043J Advanced Mathematics (Module 4; Finance)

SECTION B — GAP-FILLING QUESTIONS

Write your answers on the **Answer Book** provided.

This section is worth a total of **28** marks, with each question worth **4** marks.

7. Evaluate the integral

$$I = \int_{-\infty}^{+\infty} dx \, e^{-q^2 x^2} = \text{_____}, \quad \text{where } q > 0.$$

8. Convert the following double integral from the Cartesian coordinate system (x, y) to the polar coordinate system (ρ, θ) :

$$I = \iint_D e^x dx dy = \text{_____},$$

with D is a 2-dimensional region enclosed by three lines

$$y = 1, \quad y = x, \quad y = \sqrt{3}x.$$

(*No need to calculate the value of the integral.)

9. Convert the following triple integral from the Cartesian coordinate system (x, y, z) to the spherical coordinate system (r, ϕ, θ) :

$$I = \iiint_{\Omega} z^2 (x^2 + y^2 + z^2)^{\frac{3}{2}} dx dy dz = \text{_____}$$

with $\Omega = \left\{ (x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 \leq 4, \, z \geq 0 \right\}$. (*No need to calculate the integral value.)

BDIC1043J Advanced Mathematics (Module 4; Finance)

- 10.** Convert the following triple integral from the spherical coordinate system (r, ϕ, θ) to the cylindrical coordinate system (ρ, θ, z) :

$$I = \int_0^{2\pi} d\theta \int_0^{\frac{\pi}{4}} \sin \phi d\phi \int_0^{\frac{1}{\cos \phi}} r^4 dr = \underline{\hspace{2cm}}$$

(*No need to calculate the integral value.)

- 11.** The roots of the following polynomial equation are _____,

$$z^4 = 1 + i, \quad z \in \mathbb{C}.$$

***Requirement:** The argument of z should be kept within $(-\pi, \pi]$.

- 12.** Consider a periodic function $f(x) = \cos\left(2x - \frac{\pi}{3}\right)$ with period $T = 2\pi$. If its Fourier series reads

$$f(x) = A_0 + \sum_{n=1}^{\infty} A_n \cos nx + \sum_{n=1}^{\infty} B_n \sin nx,$$

the coefficients, A_0 , A_n and B_n ($n \in \mathbb{Z}$, $n \geq 1$), are evaluated as _____.

- 13.** Compute the integral

$$I = \int_{-\pi}^{\pi} \left(\cos x \cos 2x + \cos 2x \cos 3x + \cdots + \cos 9x \cos 10x + \cos 10x \cos x \right) dx = \underline{\hspace{2cm}}.$$

BDIC1043J Advanced Mathematics (Module 4; Finance)

SECTION C — EXTENDED ANSWER QUESTIONS

Write your answers on the **Answer Book** provided.

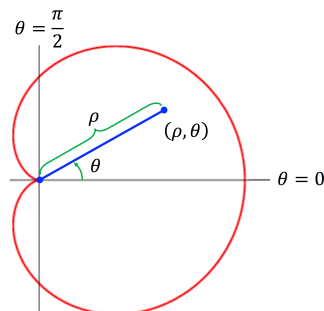
This section is worth a total of **54** marks. The marks of each question are as shown.

14. (7 marks)

Consider a *cardioid* curve in two dimensions as shown, described by the equation in polar coordinates,

$$\rho = \cos \theta + 1.$$

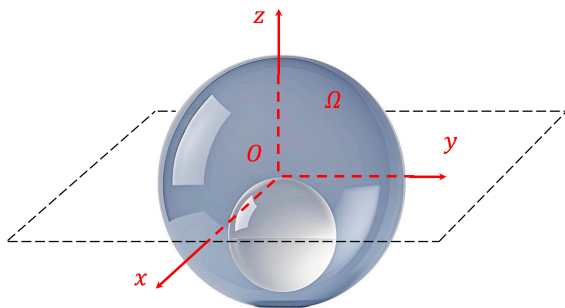
Try to compute **the area** enclosed by this curve.



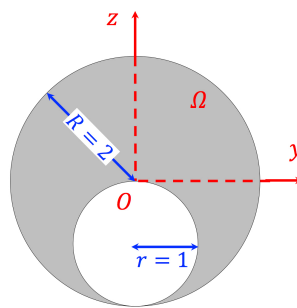
15. (11 marks) Consider a region Ω formed by two spheres as shown: (a) stereo view; (b) side view. Ω is the region outside the small sphere but inside the large sphere, where

large sphere (radius $R = 2$) : $x^2 + y^2 + z^2 = 4$;

small sphere (radius $r = 1$) : $x^2 + y^2 + (z + 1)^2 = 1$.



(a) Stereo view



(b) Side view

Try to **compute the triple integral**

$$I = \iiint_{\Omega} (x^2 + y^2 + z^2) \, dx \, dy \, dz.$$

BDIC1043J Advanced Mathematics (Module 4; Finance)

16. (9 marks) Compute the integral

$$I = \int_0^1 dx \int_x^{2x} dy \, 4y^2 e^{-y^4} + \int_1^2 dx \int_x^2 dy \, 4y^2 e^{-y^4}.$$

17. (6 marks) Use the **exponential powers** of $\sin \theta$ and $\cos \theta$ to express, separately,

$$\cos 5\theta \quad \text{and} \quad \sin 5\theta.$$

18. (9 marks) Consider a polynomial equation

$$p(z) = z^5 - z^4 + 3z^3 + 3z^2 + 4z - 10 = 0, \quad z \in \mathbb{C}.$$

If $z_1 = (1 + 2i)$ is known to be a root, then try to find the other root(s) of the equation.

19. (12 marks) (*From Fourier series to Fourier transform*)

For a periodic function, the trigonometric form of its Fourier series is given by

$$f(x) = A_0 + \sum_{n=1}^{\infty} A_n \cos(nx) + \sum_{n=1}^{\infty} B_n \sin(nx), \quad x = \omega t, \quad \omega = \frac{2\pi}{T}, \quad (1)$$

where

$$\begin{aligned} A_0 &= \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx, & A_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx) dx, \\ B_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx) dx. \end{aligned} \quad (2)$$

For an aperiodic function, its Fourier transform is given by

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega) e^{i\omega t} d\omega, \quad (3)$$

with

$$G(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt. \quad (4)$$

Give the complete derivation procedure which generalizes the periodic case of Fourier series, eqs.(1)–(2), to the aperiodic case of Fourier transform, eqs.(3)–(4).

BDIC1043J Advanced Mathematics (Module 4; Finance)

THIS PAGE IS INTENTIONALLY LEFT BLANK.

Glossary

Aperiodic function	非周期函数
Argument	(复数的) 辐角
Blocking	切块
Cardioid curve	心脏线, 心形线
Cartesian coordinates	直角坐标, 笛卡尔坐标
Coefficient	系数
Complete derivation procedure	完整推导过程
Converging lens system	凸透镜系统
Convert	转换
Cylindrical coordinate system	柱坐标系
Double integral	双重积分
Element	(体积) 元
Enclose	包围
Evaluate	计算, 赋值
Exponential power	幂次的
Filtered off	过滤掉
Finite	有限
Fourier series	傅立叶级数
Fourier transform	傅立叶变换
Frequency spectrum	频率谱
Illustration	图示
Infinitely many	无穷多个
Integrand	被积函数
Integration strategy	积分策略
Limitation behavior	极限行为

Low-frequency	低频
Necessary condition	必要条件
Optical Fourier transform	光学傅立叶变换
Original image	原始图像
Output image	输出图像
Partition	分割
Periodic function	周期函数
Polar coordinates	极坐标
Polynomial equation	多项式方程
Rectangular coordinates	直角坐标
Region	（积分）区域
Root	（方程的）根
Side view	侧视图
Signal	信号
Singular point	奇点
Slicing	切片
Spherical coordinate system	球坐标系
Standard element	标准积分微元
Stereo view	立体视图
Stripping	切条
Trigonometric form	三角形式
Triple integral	三重积分
Unbounded	无界
Upper half-ball	上半球
Well defined	良好定义的