Beijing-Dublin International College

SEMESTER II FINAL EXAMINATION – 2023/2024

Beijing-Dublin International College
BDIC1043J Maths 4 (Advanced Mathematics; Finance)

PRINCIPAL OF COLLEGE: Wenying WU
MODULE LECTURER: Xin LIU

Time Allowed: 95 minutes

Instructions for Candidates

Your answers to ALL the questions should be written on the Answer Book.

BJUT Student ID: _____ **UCD Student ID: _____**

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge：_____ (Signature)

Instructions for Invigilators

No calculators are permitted. NO dictionaries are permitted.
No rough-work paper is to be provided for candidates.

NOTE: The exam paper has **3** sections on **10** pages, with a full score of 100 marks.

SECTION A — MULTIPLE CHOICE QUESTIONS

In each question, choose **at most one** option.
Write your answers on the **Answer Book** provided.
This section is worth a total of **21** marks, with each question worth **3** marks.

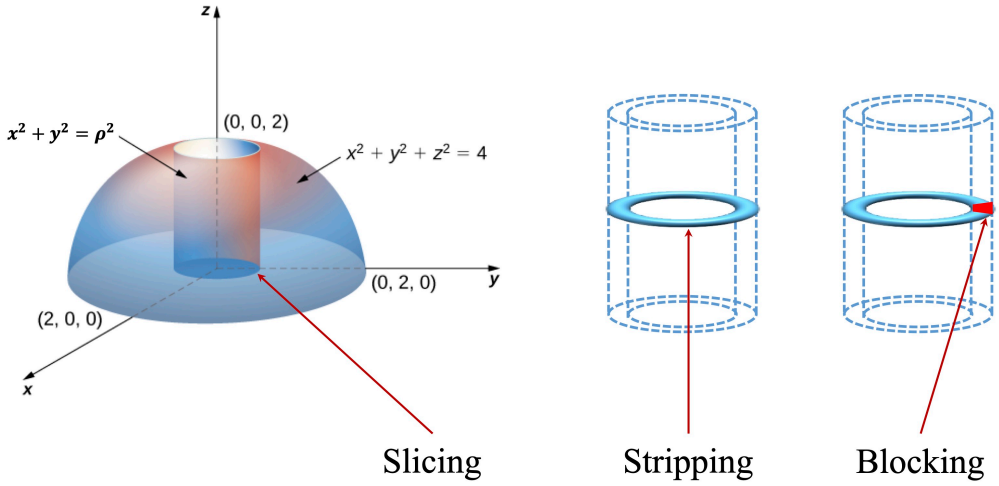
1. Consider a triple integral given in the rectangular coordinates (x, y, z),

$$I = \iiint_{\Omega} f(x, y, z)\, \mathrm{d}x\mathrm{d}y\mathrm{d}z,$$

where the integration Ω is the upper half-ball,

$$\Omega = \left\{(x, y, z) \,\middle|\, x^2 + y^2 + z^2 \leq 4, \quad z \geq 0\right\}.$$

Now let us change the coordinates to the cylindrical coordinate system, (ρ, θ, z). If the following illustration demonstrates a type of integration strategy,



then determine which of the items below describes integration strategy?

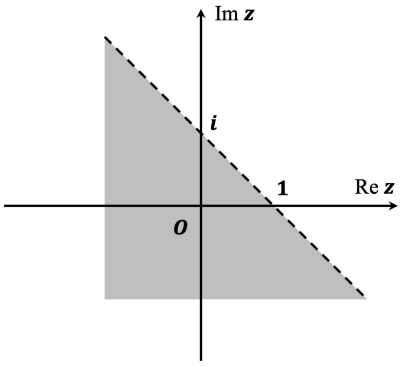
(a) $\int \mathrm{d}\rho \int \mathrm{d}\theta \int \mathrm{d}z\ f$; (b) $\int \mathrm{d}\theta \int \mathrm{d}z \int \mathrm{d}\rho\ f$; (c) $\int \mathrm{d}z \int \mathrm{d}\rho \int \mathrm{d}\theta\ f$; (d) $\int \mathrm{d}\rho \int \mathrm{d}z \int \mathrm{d}\theta\ f$.

2. Evaluate the integral

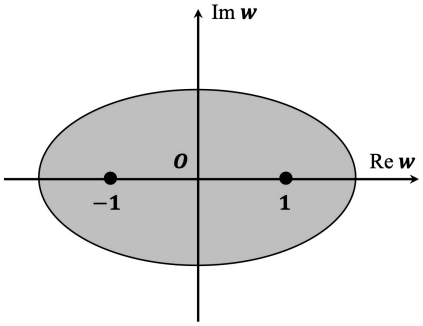
$$I = \int_0^{+\infty} \mathrm{d}x \; e^{-p^2x^2} = _____, \qquad p > 0.$$

(a) $p\sqrt{\pi}$; (b) $\frac{1}{2p}\sqrt{\pi}$; (c) $\frac{1}{2p}\pi$; (d) $p\pi$.

3. Consider two complex numbers $z, w \in \mathbb{C}$ which are described by the following illustrations.



Complex number z



Complex number w

Which item of mathematical expressions below for z and w is CORRECT?

(a) $\left\{z \in \mathbb{C} \middle| \operatorname{Im} z < -\operatorname{Re} z + 1\right\}$ and $\left\{w \in \mathbb{C} \middle| |w-1| + |w+1| \leq 4\right\}$.

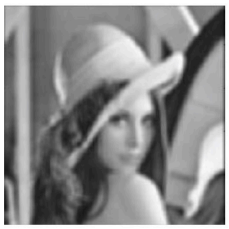
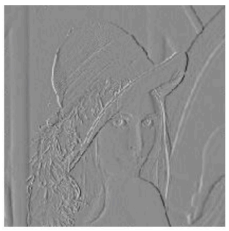
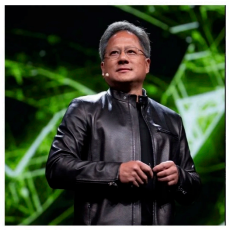
(b) $\left\{z \in \mathbb{C} \middle| \operatorname{Im} z \leq -\operatorname{Re} z + 1\right\}$ and $\left\{w \in \mathbb{C} \middle| |w-1| - |w+1| \leq 4\right\}$.

(c) $\left\{z \in \mathbb{C} \middle| |z-1| + |z+1| \leq 4\right\}$ and $\left\{w \in \mathbb{C} \middle| \operatorname{Im} w < -\operatorname{Re} w + 1\right\}$.

(d) $\left\{z \in \mathbb{C} \middle| \operatorname{Im} z < \operatorname{Re} z - 1\right\}$ and $\left\{w \in \mathbb{C} \middle| |w-1| + |w+1| \leq 4\right\}$.

4. For an original photo

the **low pass** filtered result of Fourier transform is _____below.

(a) (b)  (c)  (d) 

5. Use the exponential powers of $\sin\theta$ and $\cos\theta$ to expand $\cos 5\theta =$ _____.

(a) $\cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$;

(b) $5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta$;

(c) $\cos^5\theta + 5\cos^3\theta\sin^2\theta - 3\cos\theta\sin^4\theta$;

(d) $-3\cos^4\theta\sin\theta + 5\cos^2\theta\sin^3\theta + \sin^5\theta$.

6. For the Dirac δ-function below, compute the integral

$$\int_{-\infty}^{\infty} \cos(\omega t)\,\delta(\omega - 1)\,\mathrm{d}\omega = _____, \qquad \forall t \in \mathbb{R}.$$

(a) 0; (b) 1; (c) $\cos 2t$; (d) $\frac{e^{\mathrm{i}t}+e^{-\mathrm{i}t}}{2}$.

7. (MATLAB) Suppose array `x` is defined as `x = linspace(0,1,10);`, then which of the following expressions could extract even indices of the array only?

(a) `y = x(1:end);`

(b) `y = x(1:end/2);`

(c) `y = x(2:2:end);`

(d) `y = x(2:end-1);`

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SECTION B — GAP-FILLING QUESTIONS

Write your answers on the **Answer Book** provided.
This section is worth a total of **24** marks, with each question worth **4** marks.

8. Evaluate the integral

$$I = \int_0^1 dx \int_x^1 dy \; 3ye^{-y^3}.$$

9. Evaluate the integral

$$I = \iint_D e^x \mathrm{d}\sigma = _____,$$

where D is a region enclosed by three lines

$$y = 1, \qquad y = 2x, \qquad y = 4x.$$

10. Convert the following triple integral from the Cartesian coordinate system (x, y, z) to the spherical coordinate system (r, ϕ, θ):

$$I = \iiint_\Omega z\left(x^2 + y^2\right)^{\frac{3}{2}} \mathrm{d}x\mathrm{d}y\mathrm{d}z = _____$$

with $\Omega = \left\{(x, y, z) \middle| x^2 + y^2 + z^2 \leq 4, \quad z \geq 0\right\}$. (**No need to calculate the value of the integral.*)

11. Consider a periodic function $f(x) = \sin\left(x + \frac{\pi}{4}\right)$ with period $T = 2\pi$. If its Fourier series reads

$$f(x) = A_0 + \sum_{n=1}^{\infty} A_n \cos nx + \sum_{n=1}^{\infty} B_n \sin nx,$$

the coefficients, A_0, A_n and B_n ($n \in \mathbb{Z}$, $n \geq 1$), can be expressed as _____.

12. Compute the integral

$$I = \int_{-\pi}^{\pi} \Big(\sin x \cos 2x + \sin 2x \cos 3x + \cdots + \sin 9x \cos 10x + \sin 10x \cos x \Big) \mathrm{d}x = _____.$$

13. (MATLAB) Construct the function $y = \dfrac{x^2}{x^3 + 1}$ for values of x from 1 to 2 in steps of 0.01, and suppress any intermediate output.

The MATLAB code reads _____

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SECTION C — EXTENDED ANSWER QUESTIONS

Write your answers on the **Answer Book** provided.
This section is worth a total of **55** marks. The marks of each question are as shown.

14. (5 marks) (MATLAB) This code is used to obtain three numbers a, b and c from a user and then produce the results $a+b+c$, $\frac{a}{(b+c)(c+a)}$ and $\frac{a}{bc}$.
Try to fix errors through the lines one-by-one.

```
a = input(' Please enter a );
b = 1nput(' Please enter b );
a = Input ' Please enter c ';

v1 = a+ B+d;
v2 = a/((b+c)(c+a);
v3 = a/b*c;

disp(' a + b + c= ' int2str(v1)])
disp(['v2 = ' num2str v2 ]
disp(['v3 = num2str(v4) ])
```

15. (7 marks) Consider a polynomial equation

$$p(z) = z^5 - 10z^4 + 54z^3 - 130z^2 + 125z = 0, \qquad z \in \mathbb{C}.$$

If $z_1 = (2 + \mathrm{i})$ is known to be a root, then try to find the other root(s) of the equation.

16. (13 marks) (*From Fourier series to Fourier transform*)

For a periodic function, the trigonometric form of its Fourier series is given by

$$f(x) = A_0 + \sum_{n=1}^{\infty} A_n \cos(nx) + \sum_{n=1}^{\infty} B_n \sin(nx), \qquad x = \omega t, \qquad \omega = \frac{2\pi}{T}, \tag{1}$$

where

$$\begin{aligned} A_0 &= \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\mathrm{d}x, \qquad A_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\mathrm{d}x, \\ B_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\mathrm{d}x. \end{aligned} \tag{2}$$

For an aperiodic function, its Fourier transform is given by

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} G(\omega)e^{i\omega t}\mathrm{d}\omega, \tag{3}$$

with

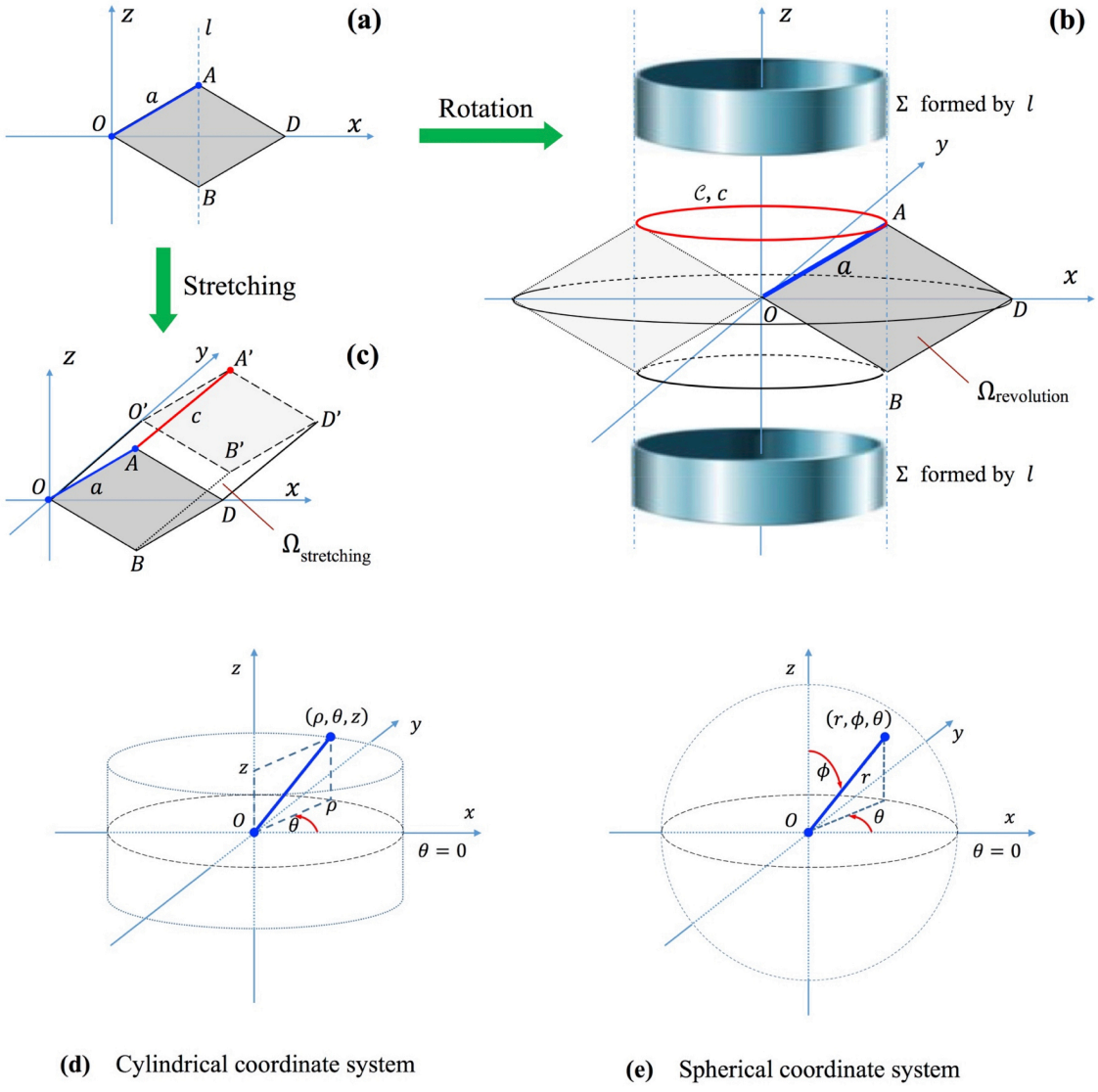
$$G(\omega) = \int_{-\infty}^{\infty} f(t)e^{-i\omega t}\mathrm{d}t. \tag{4}$$

Give the **complete derivation procedure** which generalizes the periodic case of Fourier series, eqs.(1)–(2), to the aperiodic case of Fourier transform, eqs.(3)–(4).

17. (30 marks) As shown in Sub-figure (a) below, consider a rhombus $\diamond AOBD$ in the xz-plane, formed by two regular triangles $\triangle AOB$ and $\triangle ABD$, with O being the origin of the coordinate system. The segment $\overline{AB}$ lies in a line l, and the length of an edge of the triangle is, say, $\left|\overline{OA}\right| = a$.

Generalizing the coordinate system to 3 dimensions,

- as shown in Sub-figure (b). Let $\diamond AOBD$ **rotate** around the z-axis to form a revolution body $\Omega_{\text{revolution}}$. Let the trajectory of A, which is a circle, be denoted as $\mathcal{C}$, with length c. The revolution of the line l forms a cylindrical surface, denoted as Σ.
- as shown in Sub-figure (c). Let $\diamond AOBD$ **move** along the y-direction to sweep a cylindrical body $\Omega_{\text{stretching}}$, i.e., $\diamond AOBD$-$\diamond A'O'B'D'$, with the length of a side-edge being $\left|\overline{AA'}\right| = c$, say.



(d) Cylindrical coordinate system

(e) Spherical coordinate system

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Questions:

(a) Compute the volume of $\Omega_{\text{stretching}}$. (*3 marks*)

(b) Compute the volume of $\Omega_{\text{revolution}}$, in terms of the cylindrical coordinate system, i.e., (ρ, θ, z), as shown in Sub-figure (d). (*7 marks*)

(c) Compute the volume of $\Omega_{\text{revolution}}$, in terms of the spherical coordinate system, i.e., (r, ϕ, θ), as shown in Sub-figure (e). (*12 marks*)

*Express your results of (a), (b) and (c) in terms of the parameter a (— [note]: c can be expressed by a).

(d) Compute the ratio between the two volumes,

$$\Omega_{\text{revolution}} : \Omega_{\text{stretching}}.$$

What conclusion can you draw from the ratio? (*1 marks*)

(e) In Sub-figure (b), compute the part of $\Omega_{\text{revolution}}$ inside the cylindrical surface Σ, denoted as $\Omega_{\text{revolution-inside}}$. (*4 marks*)

(f) In Sub-figure (b), compute the part of $\Omega_{\text{revolution}}$ outside the cylindrical surface Σ, denoted as $\Omega_{\text{revolution-outside}}$. Then compute the ratio,

$$\Omega_{\text{revolution-inside}} : \Omega_{\text{revolution-outside}}.$$

Which part is larger, inside or outside? (*3 marks*)

Glossary

Aperiodic function	非周期函数
Array	数组
Blocking	切块
Cartesian coordinates	直角坐标，笛卡尔坐标
Coefficient	系数
Complete derivation procedure	完整推导过程
Cylindrical coordinate system	柱坐标系
Edge	边
Enclose	包围
Errors	错误
Even	偶数
Exponential power	幂次的
Fourier series	傅立叶级数
Fourier transform	傅立叶变换
Illustration	图示
Intermediate output	中间输出
Low-pass filter	低通过滤
Origin	（坐标系）原点
Original photo	原始图片
Periodic function	周期函数
Polynomial equation	多项式方程
Ratio	比例
Rectangular coordinates	直角坐标
Regular triangle	正三角形
Revolution body	旋转体
Rhombus	菱形
Root	（方程的）根
Rotate	旋转
Segment	线段
Slicing	切片
Spherical coordinate system	球坐标系
Strategy	策略
Stripping	切条
Sub-figure	子图
Suppress	抑制，略去
Sweep	扫出
Trajectory	轨迹
Upper half-ball	上半球
Volume	体积