



Beijing-Dublin International College



SEMESTER II FINAL EXAMINATION – 2021/2022

BDIC1030J & BDIC1026J Maths 2 (Advanced Mathematics)

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Time Allowed: 90 minutes

Instructions for Candidates

Answer ALL questions. The marks that each question carry is written as shown.

BJUT Student ID: _____ **UCD Student ID:** _____

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge: _____ **(Signature)**

Instructions for Invigilators

Non-programmable calculators are permitted. NO dictionaries are permitted.
No rough-work paper is to be provided for candidates.

NOTE: Answer **ALL** questions.

Time allowed is **90** minutes.

The exam paper has **2** sections on **6** pages, with a full score of 100 marks.

You are required to use only the provided **Examination Book** for answers.

SECTION A — Brief Answer Questions

This section is worth a total of **70** marks, with each question worth **5** marks.

1. Compute the indefinite integral

$$\int \frac{x^2}{x^3 + 1} dx = \underline{\hspace{2cm}}.$$

2. Compute the indefinite integral

$$\int x^2 \ln(x) dx = \underline{\hspace{2cm}}.$$

3. Compute the indefinite integral

$$\int \sin^3(x) \cos^2(x) dx = \underline{\hspace{2cm}}.$$

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4. Compute the indefinite integral

$$\int \frac{1}{x(x-1)^2} dx = \underline{\hspace{2cm}}.$$

5. Compute the indefinite integral

$$\int \frac{x^2}{[\sqrt{1-x^2}]^3} dx = \underline{\hspace{2cm}}.$$

6. Compute the indefinite integral

$$\int \ln(x) dx = \underline{\hspace{2cm}}.$$

7. Evaluate the definite integral

$$\int_0^2 x^x (1 + \ln(x)) dx = \underline{\hspace{2cm}}.$$

8. Evaluate the definite integral

$$\int_{-1}^1 \sin(x) \cos(x) \log(x^2) e^{x^2} dx = \underline{\hspace{2cm}}.$$

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9. Evaluate the definite integral

$$\int_0^2 x \cdot \sqrt{2-x} dx = \underline{\hspace{2cm}}.$$

10. Consider an ODE

$$\frac{dy}{dx} = ky^2, \quad y(0) = 1, \quad y(1) = 2.$$

$$k = \underline{\hspace{2cm}}.$$

11. Consider an ODE

$$(x + \cos(y)) \frac{dy}{dx} = \cot(y).$$

Its general solution is given by $x(y) = \underline{\hspace{2cm}}$.

12. Consider an ODE

$$x \frac{dy}{dx} - 2y = x^4 y^2.$$

Its general solution is given by $y(x) = \underline{\hspace{2cm}}$.

13. Suppose $y^* = x \cos(x)e^{2x}$ is a particular solution of a second-order linear non-homogeneous ODE with constant coefficients. The corresponding characteristic roots are then $2 \pm i$.

14. Suppose $y = C_1 e^{-x} + C_2 x e^{-x}$ is a general solution of a second-order linear ODE with constant coefficients, C_1 and C_2 being two constants. Then a possible expression for this ODE is

$$y'' + 2y' + y = 0$$

$$(\lambda + 1)^2 = \lambda^2 + 2\lambda + 1$$

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SECTION B — Extended Answer Questions

Write your answers on the **Examination Book** provided.

This section is worth a total of **30** marks. The marks of each question are as shown.

- 15. (4 marks)** Write the following limit as a definite integral:

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(3 \left(\frac{i}{n} \right)^2 + 2 \left(\frac{i}{n} \right) + 1 \right) \left(\frac{1}{n} \right)$$

- 16. (10 marks)** Find the particular solution of the following ODE:

$$y'' - y' - 2y = (3x^2 + x + 1)e^{-x}, \quad y(0) = 0, \quad y'(0) = 0$$

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17. (16 marks) In the two-dimensional xy -plane, consider the curve specified by $\{x = x, y = 1 - x^2\}$ for $x \in [0, 1]$

- (a) Write down a mathematical expression for length of this curve using definite integration. (4 marks)
- (b) By performing the integration, calculate the length of the curve. (4 marks)
- (c) Calculate the area of the region specified by this curve and the lines $x = 0, y = 0$. (4 marks)
- (d) Find the volume of the solid body obtained by rotating this region about the x -axis. (4 marks)

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USEFUL FORMULAE

$$\int \frac{1}{\sqrt{x^2 + 1}} dx = \ln(x + \sqrt{x^2 + 1}) + c$$

$$\int \frac{1}{\cos^2 x} dx = \tan x + c$$

$$\int \frac{1}{\sin^2 x} dx = -\cot x + c$$

$$\int \tan x dx = -\ln |\cos x| + c$$

$$\int \cot x dx = \ln |\sin x| + c$$

$$\int \frac{1}{1 + x^2} dx = \arctan x + c$$

$$\int \frac{1}{x} dx = \ln x + c$$

$$\int \frac{1}{\sqrt{1 - x^2}} dx = \arcsin x + c$$