



Beijing-Dublin International College



SEMESTER I FINAL EXAMINATION – 2019/2020

**School of Mathematics and Statistics
BDIC1029J & BDIC1025J Maths 1 (Advanced Mathematics)**

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OTHER EXAMINER: Bin ZHENG

Time Allowed: 90 minutes

Instructions for Candidates

Answer ALL questions. The marks that each question carry is written as shown.

BJUT Student ID: _____ **UCD Student ID:** _____

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge: _____ **(Signature)**

Instructions for Invigilators

Non-programmable calculators are permitted. NO dictionaries are permitted.
No rough-work paper is to be provided for candidates.

NOTE: Answer **ALL** questions.

Time allowed is **90** minutes.

The exam paper has **2** sections on **4** pages, with a full score of 100 marks.

You are required to use the provided **Examination Book** only for answers.

Section A: Fill-in-the-blank Questions

This section is worth a total of **80** marks, with each question worth **5** marks.

1. Evaluate

$$\frac{\sin \frac{1}{x}}{\frac{1}{x}} = 1$$

$$\lim_{x \rightarrow \infty} x \cdot \sin \frac{1}{x} = 1$$

2. Find the limit

$$\frac{1}{x} - \frac{1}{x^2} + \frac{1}{x^3}$$

$$\lim_{x \rightarrow \infty} \frac{x^2 - x + 1}{1 - x^2 - x^3} \cdot \cos x = 0$$

3. Find the limit

$$\frac{1}{x^3} - \frac{1}{x} - 1$$

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\tan x - 1}{x - \frac{\pi}{4}} = 2$$

4. Let $f(x)$ be the function

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sec^2 x}{1} = \frac{1}{\cos^2 x} = 2$$

$$f(x) = \frac{1}{x^2 + 3x + 2}.$$

Find the higher order derivative $f^{(20)}(x) =$ _____.

5. Let $f(x)$ be the function

$$f(x) = \ln(\sqrt{x^2 + 1} + x).$$

Find the differential $df(x) =$ _____.

$$\frac{1}{\sqrt{x^2 + 1} + x} \cdot \left(\frac{1}{2} \frac{1}{\sqrt{x^2 + 1}} \cdot 2x + 1 \right)$$

6. Find the limit

$$\lim_{x \rightarrow 0} (\cos x)^{\frac{1}{x^2}} = e^{-\frac{1}{2}}$$

$$\ln y = \frac{1}{x^2} \ln \cos x$$

$$= \frac{\frac{-\sin x}{\cos x}}{2x} = \frac{-\tan x}{2x} = \frac{-\sec^2 x}{2} = -\frac{1}{2}$$

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7. Given

$$\lim_{x \rightarrow 0} \frac{e^{h(x)} - 1}{\tan x} = 5,$$

find the limit

$$\lim_{x \rightarrow 0} \frac{h(x)}{x} = \underline{5}.$$

8. Given $f(x) = \frac{\sin x}{x^2 + x}$, then the equation of the horizontal asymptote of the curve $f(x)$ is $y = \underline{0}$ whereas the equation of the vertical asymptote of the curve $f(x)$ is $x = \underline{-1}$.

9. Given $f'(1) = 3$ exists,

$$\lim_{x \rightarrow 0} \frac{f(1 + \sin 2x) - f(1)}{x} = \underline{6},$$

$$\lim_{x \rightarrow 0} \frac{f(1 + \sin 2x) - f(1)}{\sin x} \cdot \frac{\sin x}{2x} \cdot 2 = 6$$

10. Given

$$y = x \cdot \cos 2x,$$

find the higher order derivative $y^{(20)} = \underline{\hspace{2cm}}$.

11. Find the limit

$$\lim_{x \rightarrow 0} \frac{x}{\sqrt[5]{x+1} - 1} = \underline{5}.$$

12. Given

$$\begin{cases} x = \cos t, \\ y = \sin t, \end{cases} \quad \text{sin} \quad t \text{ being a parameter, } t \in \mathbb{R},$$

evaluate

$$\frac{dy}{dx} = \underline{-\frac{1}{\tan t}}, \quad \frac{d^2y}{dx^2} = \underline{\frac{1}{\sin^3 t}}.$$

13. Find $\frac{dy}{dx}$, where

$$y = \left(\frac{x}{x+1} \right)^x.$$

Advanced Mathematics (Module 1)

14. Given

$$xy + e^y + y = 2$$

then $y' = -\frac{y}{x + e^y + 1}$

15. Evaluate the values of constants a and b , such that the function

$$f(x) = \begin{cases} 1 + \sin 2x, & x \leq 0 \\ a + bx, & x > 0 \end{cases}$$

$1+0=1$
 $2\cos 2x = b$
 $a=1$

is derivable at $x = 0$.(Hint: Find $a = 1$, $b = 2$)

16. Given $f(x) = 2x^3 - 9x^2 + 12x + 1$, the monotonically decreasing interval of $f(x)$ is _____, its local maximum value is _____ and local minimum value is _____.

Advanced Mathematics (Module 1)

Section B: Extended Answer Questions

This section is worth a total of **20** marks, with each question worth **5** marks.

17. Suppose $f(x)$ is a continuous function over the interval $[0, 1]$. $f(x)$ is derivable on the point $(0, 1)$, with evaluation $f(1) = 0$.

Try to prove that there exists at least one number $\xi \in (0, 1)$, such that

$$f'(\xi) = -\frac{3f(\xi)}{\xi}.$$

18. Prove that the identity $2 \arctan x + \arcsin \frac{2x}{1+x^2} = \pi$, where $x \geq 1$.

19. Determine the limit

$$\lim_{x \rightarrow 0} \frac{x \cdot \cos x - \sin x}{x^2 \arctan x}$$

by using equivalent infinitesimal substitution theorem and L'Hospital Law.

$$= \lim_{x \rightarrow 0} \frac{x \cos x - \sin x}{x^3} = \lim_{x \rightarrow 0} \frac{\cos x + x(-\sin x) - \cos x}{3x^2} = \lim_{x \rightarrow 0} \frac{-x \sin x}{3x^2} = \lim_{x \rightarrow 0} \frac{-\sin x - x \cos x}{6x}$$

20. Show that $\ln(1+x) \geq \frac{\arctan x}{1+x}$ for $x \geq 0$.

$$\ln(1+x) - \frac{\arctan x}{1+x} \geq 0.$$

$$\text{Let } y = \ln(1+x) - \frac{\arctan x}{1+x}$$

$$y' = \frac{1}{1+x} - \frac{\frac{1}{1+x^2} \cdot (1+x) - \arctan x}{(1+x)^2}$$

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$$= \frac{1+x - \frac{1+x}{1+x^2} - \arctan x}{(1+x)^2}$$

$$= \frac{\frac{x}{1+x^2} - \arctan x}{(1+x)^2} \geq 0$$

$$y|_{x=0} = 0 - 0 = 0$$

$\therefore$ When $x \geq 0$

$$\ln(1+x) - \frac{\arctan x}{1+x} \geq 0$$

$g(x) = (1+x) - (1+x^2) \arctan x$
 $g'(x) = 1 - 2x \arctan x - (1+x^2) \frac{1}{1+x^2}$
 $= -2x \arctan x$
 $x \geq 0 \quad \therefore g'(x) \leq 0$
 $g(0) = 1 - 0 = 1$

Glossary

Curve

曲线

Decreasing

递减

Derivative

导数

Differentiable

可微分的

Differential

微分

Discontinuity

不连续

Higher order derivative

高阶导数

Horizontal asymptote

水平渐近线

Vertical asymptote

铅垂渐近线

Infinitesimal

无穷小量

Local maximum

极大值

Logarithmic differentiation

对数求导法

Local Minimum

极小值

Monotonically decreasing

单调递减

Tangent line

切线