Beijing-Dublin International College

SEMESTER I FINAL EXAMINATION - 2018/2019

School of Electrical and Electronic Engineering

EEEN3003J Signals and Systems

Dr Barry Cardiff
Armia Salib

Time Allowed: 90 minutes

Instructions for Candidates

This paper contains three (3) questions, you must answer Question 1 and any other question.

All questions carry equal marks. The distribution of marks in the right margin shown as a percentage gives an approximate indication of the relative importance of each part of the question.

Show **all details** in answer book, **not** on exam paper.

BJUT Student ID: _____ **UCD Student ID: _____**

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the punishment for violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge：_____ **(Signature)**

Instructions for Invigilators

Non-programmable calculators are permitted.

No rough-work paper is to be provided for candidates.

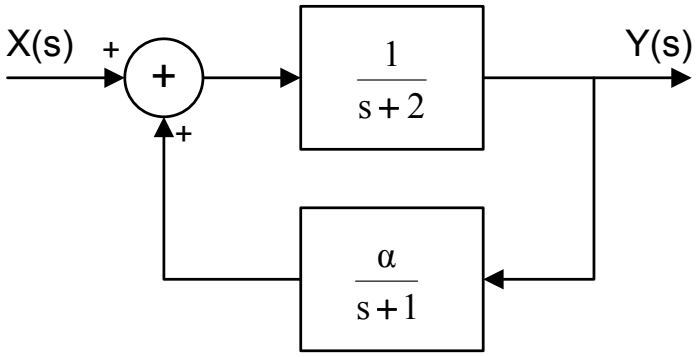
1. (i) Find the Laplace transform of the following signals (avoid integration when possible) 30%

(a) $e^{-3(t+2)}u(t)$, where $u(t)$ is the unit step function.

(b) $\sqrt{1-\sin^2(3t)}\,u(t)$

(c) $\frac{d}{dt}(te^{-t}u(t))$

(ii) For the system shown below where α is a parameter of the system,



(a) Determine the overall transfer function $H(s) = Y(s)/X(s)$. 20%

(b) Find the impulse response, $h(t)$, when $\alpha = 0$ and $\alpha = 6$. Check the stability conditions for each case. 30%

(c) Determine the range of α that makes this system stable. 20%

2. One complete period from $-\pi \leq t < \pi$ of the square wave signal $f(t)$ is given as follows

$$f(t) = \begin{cases} -1 & \text{for} \quad -\pi \leq t < 0 \\ +1 & \text{for} \quad 0 \leq t < \pi \end{cases}.$$

(i) Express this periodic function as a complex Fourier series, in the form 60%

$$f(t) = \sum_{k=-\infty}^{\infty} c_k\, e^{jk\omega_o t}.$$

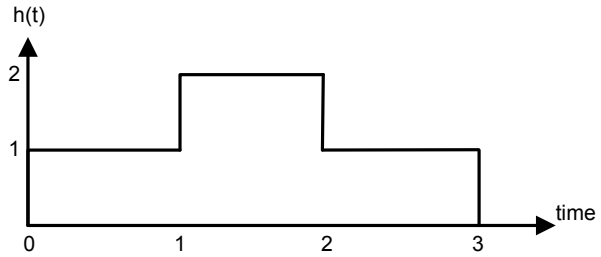
(ii) Using your answer from (i), find the trigonometric Fourier coefficients a_k and b_k for this signal, and hence express $f(t)$ as a trigonometric Fourier series. 25%

(iii) Why all a_k obtained in (ii) are equal? 15%

3. (i) Use the Laplace transform to find the zero-state response $y(t) = y_{zs}(t)$ of the following differential equation. 30%

$$\frac{d^2}{dt^2}x(t) + 3\frac{d}{dt}x(t) + 2x(t) = \sin(t)\,u(t)$$

(ii) Evaluate $y(t) = u(t) * h(t)$, where $h(t)$ is as shown below,



(a) by using the convolution formula directly, and 25%

(b) by using the Laplace transform. 20%

(c) Sketch $y(t)$ resulted from either method. 25%

Useful Formulae

Fourier Series

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\, e^{-jn\omega_0 t}\, dt, \quad \omega_0 = 2\pi/T$$

$$f(t) = c_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)$$

$$a_n = 2\mathrm{Re}(c_n),\ b_n = -2\mathrm{Im}(c_n)$$

Fourier Transform

$$F(j\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(j\omega)\, e^{j\omega t}\, dt$$

Transform Pairs

$f(t)$	$F(j\omega)$
$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$
$u(t)$	$\pi\delta(\omega)+1/(j\omega)$

Properties

$f(at)$	$\frac{1}{\lvert a\rvert}F\left(j\frac{\omega}{a}\right)$
$f(t-a)$	$e^{-j\omega a}F(j\omega)$
$\frac{d}{dt}f(t)$	$j\omega F(j\omega)$
$F(t)$	$2\pi f(-j\omega)$

Laplace Transform

$$F(s) = \int_{0^-}^{\infty} f(t)\, e^{-st}\, dt$$

Transform Pairs

$f(t)$	$F(s)$
$\sin(at)$	$\frac{a}{s^2+a^2}$
$\cos(at)$	$\frac{s}{s^2+a^2}$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\delta(t)$	1

Properties

$af(t)$	$aF(s)$
$f(at)$	$\frac{1}{a}F\left(\frac{s}{a}\right)$
$e^{-at}f(t)$	$F(s+a)$
$\int_0^t f(\tau)d\tau$	$F(s)/s$
$\frac{d}{dt}f(t)$	$sF(s)-f(0)$
$(-t)f(t)$	$\frac{d}{ds}F(s)$

Trigonometric

$$\cos(A\pm B) = \cos(A)\cos(B) \mp \sin(A)\sin(B)$$

$$\sin(A\pm B) = \sin(A)\cos(B) \pm \cos(A)\sin(B)$$

$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

$$e^{-j\theta} = \cos(\theta) - j\sin(\theta)$$

Integration

$$\int \cos(x) = \sin(x)$$

$$\int \sin(x) = -\cos(x)$$

$$\int u\,dv = uv - \int v\,du$$