



# Beijing-Dublin International College



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## AUTUMN TRIMESTER FINAL EXAMINATION - 2019/2020

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EEEN3003J: Signals and Systems

**MODULE COORDINATOR:** Dr. Barry Cardiff

**Time Allowed:** 2 Hours

### Instructions for Candidates

Answer any **three** of the 4 questions. All questions carry equal marks.

The percentages in the right margin give an approximate indication of the relative importance of each part of the particular question.

All rough work must be entered in your answer books.

**BJUT Student ID:** \_\_\_\_\_ **UCD Student ID:** \_\_\_\_\_

I have read and clearly understand the examination rules of both Beijing University of Technology and University College Dublin. I am aware of the punishment for violating the rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

**Honesty Pledge:** \_\_\_\_\_ **(Signature)**

### Instructions for Invigilators

Non-programmable calculators are permitted.

No rough-work paper is to be provided to candidates

## Question: 1

One complete period from  $-\pi \leq t < \pi$  of the square wave signal,  $f(t)$ , is given as follows:

$$f(t) = \begin{cases} -1 & \text{for } -\pi \leq t < 0 \\ +1 & \text{for } 0 \leq t < \pi \end{cases}$$

1.1 Express this function as a complex Fourier series in the form:

$$f(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_o t}$$

your answer should contain a complete derivation of the  $c_k$  terms and give a clear definition of  $\omega_o$  in the above equation. [60%]

1.2 Express the same function,  $f(t)$ , as a trigonometric Fourier series in the form:

$$f(t) = \mu + \sum_{k=1}^{\infty} a_k \cos(k\omega_o t) + b_k \sin(k\omega_o t)$$

your answer should contain a complete derivation of the  $a_k$  and  $b_k$  terms, or you may, if you want, derive these based on the  $c_k$  in part 1.1 - in this case you must prove the relationship between the trigonometric Fourier series and the complex Fourier series coefficients. [25%]

1.3 Many of the coefficients obtained in part 1.2 are zero - what general principle does this illustrate?

(Note: If you were unable to observe this effect in part 1.2 you may still attempt this part by saying how you'd expect the trigonometric Fourier series coefficients to be based on some general feature of  $f(t)$ ). [15%]

## Question: 2

If  $F(j\omega)$  and  $G(j\omega)$  is the Fourier Transform of  $f(t)$  and  $g(t)$  respectively, i.e.:

$$f(t) \xleftrightarrow{\mathcal{F}} F(j\omega) \quad \text{and} \quad g(t) \xleftrightarrow{\mathcal{F}} G(j\omega)$$

prove the following properties:

2.1  $\alpha f(t) + \beta g(t) \xleftrightarrow{\mathcal{F}} \alpha F(j\omega) + \beta G(j\omega)$  for any set of constants  $\alpha$  and  $\beta$  [20%]

2.2  $f(t - \tau) \xleftrightarrow{\mathcal{F}} e^{-j\tau\omega} F(j\omega)$  [10%]

2.3  $f(\lambda t) \xleftrightarrow{\mathcal{F}} \frac{1}{|\lambda|} F(j\frac{\omega}{\lambda})$  for any constant  $\lambda$ . Hint: be careful with the sign when doing variable substitution ( $\lambda$  can be positive or negative) [10%]

2.4 If  $f(t)$  is real, then  $F(j\omega) = F^*(-j\omega)$  [10%]

2.5 Consider the signal:

$$f(t) = A \sin(\omega_o t + \phi)$$

Derive, from first principles, the Fourier Transform of  $f(t)$ . You will get 0% for just quoting the answer as it is easily available in the 'useful formula' at the end of this paper! [25%]

2.6 Consider a system having Transfer Function (in the Fourier domain) of:

$$H(j\omega) = \frac{1}{1 + j\omega}$$

Compute its output when the input is  $A \sin(\omega_o t + \phi)$ . [25%]

## Question: 3

3.1 Find the partial fraction expansion of:

$$F(s) = \frac{1}{s(s-1)(s+1)^2}$$

Don't just give the answer - show your proof. [40%]

3.2 Hence or otherwise compute  $f(t) = \mathcal{L}^{-1}\{F(s)\}$ . [20%]

Consider the following ODE:

$$\frac{d^3}{dt^3}g(t) + 3\frac{d^2}{dt^2}g(t) + 4\frac{d}{dt}g(t) + 2g(t) = f(t)$$

3.3 What is its transfer function?

Hint: the denominator should have at least one root at  $s = -1$ ; this should help to simplify the answer. [20%]

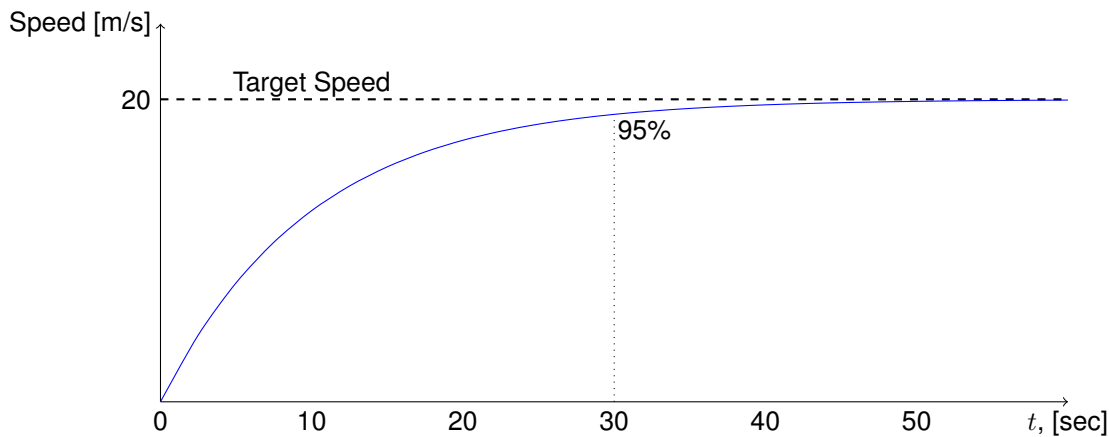
3.4 If the forcing function  $f(t)$  for the same ODE is just the unit step, i.e.  $f(t) = u(t)$  compute the Laplace domain function  $G(s)$ . [10%]

3.5 Hence or otherwise compute the time domain function  $g(t)$ .

Hint, the answer is related to part 3.1 of this question. [10%]

## Question: 4

4.1 The Figure below shows the open-loop response,  $y_{\text{open}}(t)$ , of a car's speed to an 0.001 litres/sec step input of fuel on a flat road on a calm day (not windy).



Assuming a first order response:

- Estimate the parameters of the response from the graph
- Write an equation for the car's speed,  $y_{\text{open}}(t)$ , as a function of time.
- Give the Laplace domain transfer function (speed vs fuel) of the engine  $G_p(s)$ . [20%]

Assuming an ideal speed sensor is available we'd like to use a PI controller  $G_c(s)$  in a unity feedback arrangement to control the car's speed:

4.2 Provide an expression for  $G_c(s)$  using standard notation. [20%]

4.3 Provide an expression for the closed loop transfer function of the whole system in terms of the parameters of the PI controller. [20%]

4.4 What is the maximum value of  $k_i$  one could use (in terms of  $k_d$ ) such that the system is stable? (explain your answer and show your proofs). [20%]

4.5 Using a value of  $k_p = \frac{5}{20000}$ , select a value of  $k_i$  such that:

- The system is stable
- The response has no oscillations
- The response converges quicker than for any other choice of  $k_i$ . [20%]

### Fourier Series

$$c_k = \frac{1}{T} \int_T f(t) e^{jk\omega_o t} dt, \quad \omega_o = \frac{1}{T}$$

$$f(t) = \mu + \sum_{k=1}^{\infty} a_k \cos(k\omega_o t) + b_k \sin(k\omega_o t)$$

$$a_k = 2\Re\{c_k\} \quad , \quad b_k = -2\Im\{c_k\}$$

### Fourier Transform

$$F(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(j\omega) e^{j\omega t} d\omega$$

### Laplace Transform Pairs

$$f(t) \xleftrightarrow{\mathcal{L}} F(s)$$

$$\sin(at) \xleftrightarrow{\mathcal{L}} \frac{a}{s^2 + a^2}$$

$$\cos(at) \xleftrightarrow{\mathcal{L}} \frac{s}{s^2 + a^2}$$

$$t^n \xleftrightarrow{\mathcal{L}} \frac{n!}{s^{n+1}}$$

$$e^{at} \xleftrightarrow{\mathcal{L}} \frac{1}{s - a}$$

$$\delta(t) \xleftrightarrow{\mathcal{L}} 1$$

$$1 \xleftrightarrow{\mathcal{L}} \frac{1}{s}$$

### Trigonometric Formula

$$\cos(A \pm B) = \cos(A) \cos(B) \mp \sin(A) \sin(B)$$

$$\sin(A \pm B) = \sin(A) \cos(B) \pm \cos(A) \sin(B)$$

$$e^{j\theta} = \cos(\theta) + j \sin(\theta)$$

### Fourier Transform Pairs

$$e^{j\omega_o t} \xleftrightarrow{\mathcal{F}} 2\pi\delta(\omega - \omega_o)$$

$$u(t) \xleftrightarrow{\mathcal{F}} \pi\delta(\omega) + \frac{1}{j\omega}$$

### Fourier Properties

$$f(\lambda t) \xleftrightarrow{\mathcal{F}} \frac{1}{|\lambda|} f\left(j\frac{\omega}{a}\right)$$

$$f(t - \tau) \xleftrightarrow{\mathcal{F}} e^{-j\omega\tau} F(j\omega)$$

$$\frac{d}{dt} f(t) \xleftrightarrow{\mathcal{F}} j\omega F(j\omega)$$

### Laplace Properties

$$f(at) \xleftrightarrow{\mathcal{L}} \frac{1}{a} f\left(j\frac{s}{a}\right)$$

$$e^{-at} f(t) \xleftrightarrow{\mathcal{L}} F(s + a)$$

$$\int_0^t f(\tau) d\tau \xleftrightarrow{\mathcal{L}} \frac{1}{s} F(s)$$

$$\frac{d}{dt} f(t) \xleftrightarrow{\mathcal{L}} s f(s) - f(0)$$

$$\frac{d^n}{dt^n} f(t) \xleftrightarrow{\mathcal{L}} s^n F(s) - \sum_{k=1}^n s^{n-k} \frac{d^{k-1}}{dt^{k-1}} f(t) \Big|_{t=0^-}$$

$$(-t) f(t) \xleftrightarrow{\mathcal{L}} \frac{d}{ds} F(s)$$

### Integrals

$$\int \cos(x) = \sin(x)$$

$$\int \sin(x) = -\cos(x)$$

$$\int u dv = uv - \int v du$$

### Partial fractions

$$\text{If } F(s) = \frac{N(s)}{\prod_i (s - p_i)^{k_i}} = \sum_i \left\{ \frac{r_{i,1}}{s - p_i} + \frac{r_{i,2}}{(s - p_i)^2} + \dots + \frac{r_{i,k_i}}{(s - p_i)^{k_i}} \right\}$$

$$\text{then } r_{i,j} = \frac{1}{(k_i - j)!} \lim_{s \rightarrow p_i} \frac{d^{k_i-j}}{ds^{k_i-j}} (s - p_i)^{k_i} F(s)$$

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