



北京工业大学北京 – 都柏林国际学院
BEIJING-DUBLIN INTERNATIONAL COLLEGE AT BJUT

SEMESTER 2 RESIT EXAMINATIONS
ACADEMIC YEAR 2022/2023

EEEN3018J Numerical Methods and Optimization

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I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge: _____ **(Signature)**

Time Allowed: 2 HOURS

Instructions for Candidates

This examination paper contains **FIVE** questions.

Answer question 1 **AND EITHER** question 2 or question 3, **AND EITHER** question 4 or question 5 for a maximum of 200 marks.

Questions 2 and 3 carry equal marks. Questions 4 and 5 carry equal marks.

The distribution of marks for the parts of each question is indicated in () brackets

Instructions for Invigilators

Non-programmable calculators are permitted.

No rough-work paper is to be provided for candidates.

Formulas

These formulas are provided without further context.

$$f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{(x_n - x_{n-1})f(x_n)}{f(x_n) - f(x_{n-1})}$$

$$\begin{pmatrix} \sum_{i=1}^n x_i^2 & \sum_{i=1}^n x_i \\ \sum_{i=1}^n x_i & n \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^n x_i y_i \\ \sum_{i=1}^n y_i \end{pmatrix}$$

$$A^T A c = A^T y$$

$$a_i = \frac{y_{i+1} - y_i}{x_{i+1} - x_i}$$

$$b_i = y_i$$

$$\begin{bmatrix} \frac{2h_0}{3} & \frac{h_0}{3} & 0 & \dots & 0 & 0 & 0 \\ \frac{h_0}{3} & \frac{2}{3}(h_1 + h_0) & \frac{h_1}{3} & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & \frac{h_{n-2}}{3} & \frac{2}{3}(h_{n-2} + h_{n-1}) & \frac{h_n}{3} \\ 0 & 0 & 0 & \dots & 0 & \frac{h_{n-1}}{3} & \frac{2h_{n-1}}{3} \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_{n-1} \\ b_n \end{bmatrix} = \begin{bmatrix} \frac{f_0}{h_0} - \alpha \\ \frac{f_1}{h_1} - \frac{f_0}{h_0} \\ \vdots \\ \frac{f_{n-1}}{h_{n-1}} - \frac{f_{n-2}}{h_{n-2}} \\ \beta - \frac{f_{n-1}}{h_{n-1}} \end{bmatrix}$$

$$hf\left(\frac{h}{2}\right)$$

$$h[f(0) + f(h)]$$

$$\frac{h}{3}[f(0) + 4f(h) + f(2h)]$$

$$x_{n+1} = x(t_n) + \Delta t f(t_n, x_n)$$

$$x_{n+1} = x_n + \frac{1}{2}\Delta t [f(t_n, x_n) + f(t_n + \Delta t, x_{n+1}^p)]$$

$$k_1 = \Delta t f(t_n, x_n)$$

$$k_2 = \Delta t f(t_n + \alpha \Delta t, x_n + \beta k_1)$$

$$x_{n+1} = x_n + ak_1 + bk_2$$

Q 1

a. Convert the following program to Standard Equality Form:

$$\begin{array}{ll}
 \min & z(x) = -9x_1 + 2x_2 - 2x_3 \\
 \text{s.t.} & -3x_1 + x_2 \geq -14 \\
 & 2x_1 + x_2 - x_3 \leq 3 \\
 & 6x_2 + 2x_3 = 3 \\
 & x_1, x_3 \geq 0
 \end{array}$$

(10 marks)

b. A manufacturer wants to optimise their production in order to maximise their profit. They produce writing paper and poster paper. The sale price on a box of writing paper is €30 while the sale price on a box of poster paper is €25. The products are manufactured with a single machine. Each box of writing paper requires 11 kg of raw pulp to produce while poster paper requires 6 kg per box. The daily supply of raw pulp is limited to at most 800 kg at a cost of €0.15 per kg. 12 boxes of writing paper can be produced in an hour. 15 boxes of poster paper can be produced per hour. The machine can be in operation for a full work day (8 hours). Create a linear optimisation problem which describes the systems production over a single day so that an optimal operation point can be decided.

(10 marks)

c. In your own words, and with the use of a diagram, explain what is meant by a “certificate of unboundedness”, what is required for a certificate of unboundedness, and how does this prove an unbounded problem.

(10 marks)

d. Consider the LP problem:

$$\max\{c^T x : Ax = b, x \geq 0\}, \text{ where}$$

$$A = \begin{pmatrix} 3 & 2 & -4 & 2 \\ 1 & 3 & 2 & 3 \end{pmatrix} \quad b = \begin{pmatrix} 4 \\ 6 \end{pmatrix} \quad c^T = (-2, 3, 4, -3) \quad B = \{2, 3\}$$

write the LP problem in canonical form for the Basis B.

(10 marks)

e. From the given function:

$$f(x) = x_3^2 - 4x_2^2 + 3x_1^2 - 2x_3x_2 + 7x_3x_1 - x_3 - 5x_2 - 2x_1 + 12$$

Determine if the function is strictly convex.

(Hint: first determine the Hessian matrix(H), $\nabla^2 F$)

(10 marks)

(Total: 50 marks)

Q 2

From the following optimisation problem:

$$\begin{aligned} \max \quad & z(x) = (4, 8, 0, 0)x \\ \text{s.t.} \quad & \begin{pmatrix} -3 & 2 & 1 & 0 \\ 2 & 4 & 0 & 1 \end{pmatrix} x \leq \begin{pmatrix} -1 \\ 4 \end{pmatrix} \\ & x \geq 0 \end{aligned}$$

Use the simplex algorithm to find the optimal solution x^* and optimal value $z(x^*)$ for the problem.

- Use Bland's rule to decide new entries to the Basis set.

(Total: 50 marks)

Q 3

For the following function:

$$f(x, y) = 3x^2 + 6y^2 + 3xy - 4x - 2y - 6$$

Find the point $[x, y]'$ derived after 2 iterations using Gradient Decent - Steepest decent search from the point:

$$[x, y] = [3, 2]$$

What is the location of this point $[x,y]'$?
what is the function value at the point $[x,y]'$?
Give your answer up to 3 decimal places.

Note:
For each line search work out the minimum point analytically.

(Total: 50 marks)

Q 4

- a.** State the formula for converting a normal 32-bit IEEE floating point number to decimal representation. (20 marks)
- b.** How is a denormal number represented in the 32-bit IEEE floating point format? What is the purpose of denormal numbers? (20 marks)
- c.** How do we store ∞ in 32-bit IEEE floating point format? (20 marks)
- d.** In 32-bit IEEE floating point format, what is the result of the calculation $(2^{24} + 1) - 2^{24}$? Explain your reasoning. (20 marks)
- e.** Write a short piece of MATLAB code to calculate the roots of the following function. MATLAB has the function `poly()` which can solve this, but please write your own solution.

$$\frac{x^2}{2} + 2^{12}x - \frac{1}{2} = 0$$

Identify any challenges you may encounter. (20 marks)

(Total: 100 marks)

Q 5

- a.** What does it mean to say that a curve is the least squares fit to a set of data points, $\{x_i, y_i\}$? (15 marks)
- b.** Given a set of points (x_i, y_i) , $i \in \{1, 2, \dots, n\}$ and supposing that we wish to fit a linear curve $y(x) = \alpha x + \beta$, prove that the least squares fit is given by the solution to the system of equations

$$\begin{pmatrix} \sum_{i=1}^n x_i^2 & \sum_{i=1}^n x_i \\ \sum_{i=1}^n x_i & n \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^n x_i y_i \\ \sum_{i=1}^n y_i \end{pmatrix} \quad (1)$$

(30 marks)

c. Part of a diode's V-I characteristic is given in Tab. 1. Find the least squares linear fit to these samples.

Hence, find the values of the resistor and the voltage source in the piece-wise linear diode model, as depicted in Fig. 1. (40 marks)

Table 1: V-I characteristic of the diode to be approximated.

v	1.7	1.72	1.74	1.76	1.78	1.8
i	0.002	0.0035	0.007	0.008	0.011	0.014

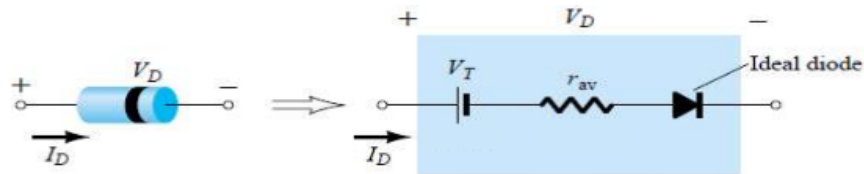


Figure 1: A diode can be approximated as a series combination of a voltage source, a resistor and an ideal diode for a piece-wise linear V-I characteristic.

d. How is least squares estimation different from interpolation? (15 marks)

(Total: 100 marks)

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