北京工业大学北京－都柏林国际学院
BEIJING-DUBLIN INTERNATIONAL COLLEGE AT BJUT

SEMESTER 2 SAMPLE PAPER EXAMINATIONS

ACADEMIC YEAR 2021/2022

EEEN3018J Numerical Methods and Optimization

Prof. Thomás Ward
Prof. John T. Sheridan
Dr John Healy
Dr Declan Delaney*

BJUT Student ID: _____ UCD Student ID: _____

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge: _____(Signature)

Time Allowed: 2 HOURS

Instructions for Candidates

This examination paper contains **FOUR** questions.
Answer **EITHER** question 1 or question 2, **AND EITHER** question 3 or question 4 for a maximum of 200 marks.
All four questions carry equal marks.
The distribution of marks for the parts of each question is indicated in ( ) brackets

Instructions for Invigilators

Non-programmable calculators are permitted.
No rough-work paper is to be provided for candidates.

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Formulas

These formulas are provided without further context.

$f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + ...$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_{n+1} = x_n - \frac{(x_n - x_{n-1})f(x_n)}{f(x_n) - f(x_{n-1}}$

$$\begin{pmatrix} \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i & n \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{n} x_i y_i \\ \sum_{i=1}^{n} y_i \end{pmatrix}$$

$A^T A c = A^T y$

$a_i = \frac{y_{i+1} - y_i}{x_{i+1} - x_i}$

$b_i = y_i$

$$\begin{bmatrix} \frac{2h_0}{3} & \frac{h_0}{3} & 0 & \cdots & 0 & 0 & 0 \\ \frac{h_0}{3} & \frac{2}{3}(h_1 + h_0) & \frac{h_1}{3} & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \frac{h_{n-2}}{3} & \frac{2}{3}(h_{n-2} + h_{n-1}) & \frac{h_n}{3} \\ 0 & 0 & 0 & \cdots & 0 & \frac{h_{n-1}}{3} & \frac{2h_{n-1}}{3} \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_{n-1} \\ b_n \end{bmatrix} = \begin{bmatrix} \frac{f_0}{h_0} - \alpha \\ \frac{f_1}{h_1} - \frac{f_0}{h_0} \\ \vdots \\ \frac{f_{n-1}}{h_{n-1}} - \frac{f_{n-2}}{h_{n-2}} \\ \beta - \frac{f_{n-1}}{h_{n-1}} \end{bmatrix}$$

$hf(\frac{h}{2})$

$h[f(0) + f(h)]$

$\frac{h}{3}[f(0) + 4f(h) + f(2h)]$

$x_{n+1} = x(t_n) + \Delta t f(t_n, x_n)$

$x_{n+1} = x_n + \frac{1}{2}\Delta t[f(t_n, x_n) + f(t_n + \Delta t, x_{n+1}^p)]$

$k_1 = \Delta t f(t_n, x_n)$

$k_2 = \Delta t f(t_n + \alpha \Delta t, x_n + \beta k_1)$

$x_{n+1} = x_n + ak_1 + bk_2$

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Q 1

a. What does it mean to say that a curve is the least squares fit to a set of data points, $\{x_i, y_i\}$? (20 marks)

b. Given a set of points (x_i, y_i), $i \in \{1, 2, ...n\}$ and supposing that we wish to fit a linear curve $y(x) = \alpha x + \beta$, prove that the least squares fit is given by the solution to the system of equations

$$\begin{pmatrix} \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i & n \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{n} x_i y_i \\ \sum_{i=1}^{n} y_i \end{pmatrix} \quad (1)$$

(30 marks)

c. Part of a diode's V-I characteristic is given in Tab. 1. Find the least squares linear fit to these samples.
Hence, find the values of the resistor and the voltage source in the piece-wise linear diode model, as depicted in Fig. 1. (40 marks)

Table 1: V-I characteristic of the diode to be approximated.

v	1.7	1.72	1.74	1.76	1.78	1.8
i	0.0019	0.0035	0.0066	0.0078	0.0108	0.0143

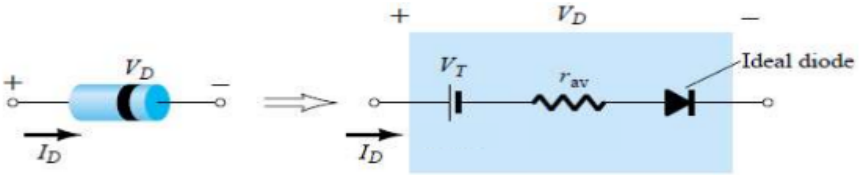

Figure 1: A diode can be approximated as a series combination of a voltage source, a resistor and an ideal diode for a piece-wise linear V-I characteristic.

d. How is least squares estimation different from interpolation? (10 marks)

(Total: 100 marks)

Q 2

The following three points lie on a quadratic curve $y = c_2x^2 + c_1x + c_0$: (1,5), (3,12), (5,9).

a. Construct a system of equations to find c_0, c_1, and c_2, writing it as a matrix product in the form $A\chi = b$. (15 marks)

b. Find the LU decomposition of the system matrix and hence solve for the values of c_0, c_1, and c_2. (25 marks)

c. Sketch the quadratic curve between $0 \leq x \leq 6$. (10 marks)

d. Calculate the area under the curve by any numerical means you have studied in this module. You should name the method you have used. (25 marks)

e. Solve for a root of the quadratic using two iterations of Newton's method, with an initial guess of 1. (25 marks)

(Total: 100 marks)

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