



Beijing-Dublin International College



SEMESTER II FINAL EXAMINATION – 2020/2021

**Beijing-Dublin International College
BDIC1043J Maths 4 (Advanced Mathematics; Finance)**

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MODULE LECTURER: Xin Liu

Time Allowed: 90 minutes

Instructions for Candidates

Answer ALL questions. The marks that each question carry is written as shown.
Write **ALL** your answers on the **EXAMINATION BOOK** provided.

BJUT Student ID: _____ **UCD Student ID:** _____

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge: _____ **(Signature)**

Instructions for Invigilators

Non-programmable calculators are permitted. NO dictionaries are permitted.
No rough-work paper is to be provided for candidates.

SECTION A — MULTIPLE CHOICE QUESTIONS

In each question, choose **at most one** option.

This section is worth a total of **30** marks, with each question worth **3** marks.

1. A triple integral over a region Ω is defined as

$$I = \iiint_{\Omega} f(x, y, z) dv = \lim_{v_i \rightarrow 0} \sum_{i=1}^n f(x_i, y_i, z_i) \Delta v_i,$$

where $\{\Delta v_1, \dots, \Delta v_i, \dots, \Delta v_n\}$ is a partition of Ω , with $i = 1, \dots, n$.

Which of the following statements is CORRECT?

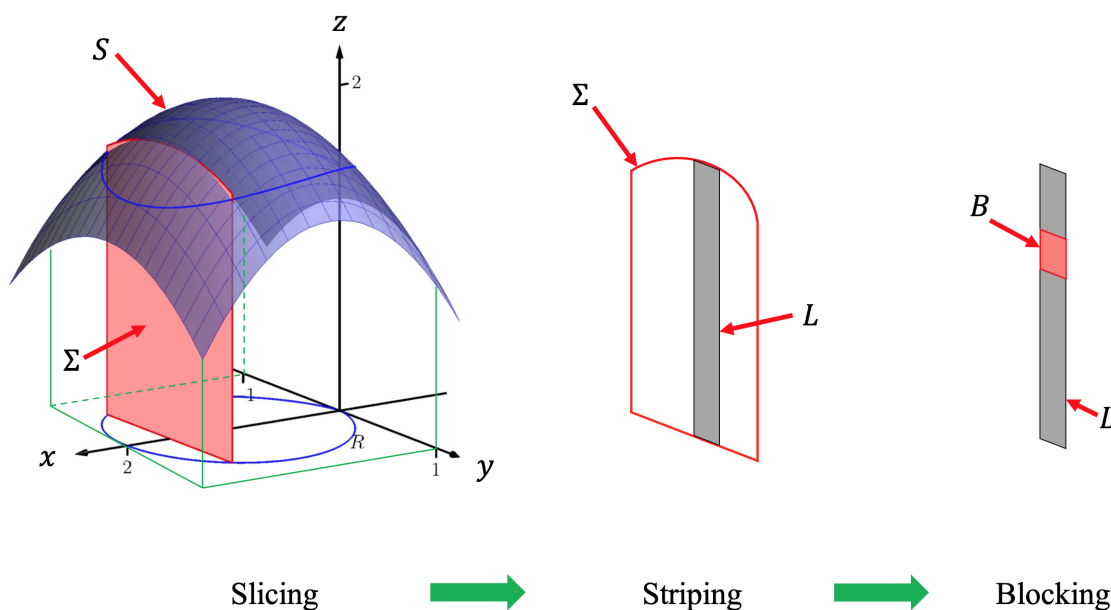
- (a) The integral I is well defined (i.e., having a finite value), even if the integrand $f(x, y, z)$ is unbounded at finite points within the region Ω .
- (b) The integral I must be well defined (i.e., having a finite value), even if the region Ω contains infinitely many singular points. *continuous,*
- (c) For this definition of integration there exists a requirement that all the small regions in the partition must be equal to each other, and equal to a standard element Δv ; i.e., $\Delta v_1 = \dots = \Delta v_i = \dots = \Delta v_n = \Delta v$, and the Δv goes to 0 in the limitation behavior.
- (d) None of the above.

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2. Consider a triple integral in the (x, y, z) -coordinate system,

$$I = \iiint_{\Omega} f(x, y, z) dv.$$

Ω is the region enclosed by the planes $x = 0$, $x = 2$, $y = -1$, $y = 1$, $z = 0$ and the surface S .



In order to perform the integration let us use the strategy:

$$\Omega \xRightarrow{\text{slicing}} \text{slice } \Sigma \xRightarrow{\text{striping}} \text{stripe } L \xRightarrow{\text{blocking}} \text{tiny block } B.$$

The above illustrated strategy corresponds to the following order of integration: _____.

- (a) $\int dx \int dy \int dz$; (b) $\int dz \int dy \int dx$; (c) $\int dx \int dz \int dy$; (d) $\int dz \int dx \int dy$.

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3. Consider a triple integral in the (x, y, z) -coordinate system,

$$I = \iiint_V x \, dv,$$

where V denotes a region satisfying

$$1 \leq x^2 + y^2 + z^2 \leq 8, \quad y \leq 0, \quad z \geq 0.$$

If being rewritten in the spherical coordinates, (r, ϕ, θ) , the integral is expressed as

- (a) $\int_1^2 dr \, r^2 \int_0^\pi d\phi \, \sin \phi \int_0^{2\pi} d\theta$;
 (b) $\int_0^8 dr \, r^3 \int_0^\pi d\phi \, \cos \phi \int_0^{2\pi} d\theta$;
 (c) $\int_1^2 dr \, r^3 \int_0^{\frac{\pi}{2}} d\phi \, \sin^2 \phi \int_\pi^{2\pi} d\theta \, \cos \theta$;
 (d) $\int_1^2 dr \, r^3 \int_{\frac{\pi}{2}}^\pi d\phi \, \sin^2 \phi \int_0^\pi d\theta \, \sin \theta$.

4. Compute the integral of **Question 3**. The result is _____.

- (a) 0; (b) $\frac{\pi}{3}$; (c) $\frac{7\pi}{3}$; (d) $-\frac{7\pi}{3}$.

5. Compute the double integral

$$I = \iint_D (x + y) dx dy,$$

where D is the region enclosed by three lines, $x = 0$, $y = 0$ and $x + y = 1$.

- (a) $\frac{1}{2}$; (b) $\frac{1}{6}$; (c) $\frac{2}{3}$; (d) $\frac{1}{3}$.

6. Compute the triple integral

$$I = \iiint_\Omega 3z \, dv,$$

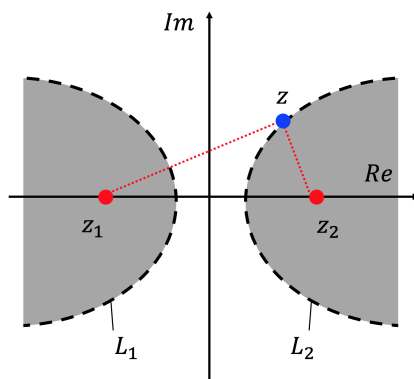
where Ω is the region below the sphere $x^2 + y^2 + z^2 = 1$ and inside the cone $z = \sqrt{x^2 + y^2}$.

- (a) $\frac{\pi}{8}$; (b) $\frac{3\pi}{8}$; (c) $\frac{3\pi}{8} + 1$; (d) $\frac{\pi}{3} (2 - \sqrt{2})$.

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7. Consider two points $z_1(-1, 0)$ and $z_2(1, 0)$ in the complex plane, as shown. Let L_1 and L_2 be the two branches of a hyperbola, as denoted by two dashed curves.

Here the *hyperbola* means: an arbitrary point z on L_1 and L_2 has a property that the difference between the distance $|z - z_1|$ and the distance $|z - z_2|$ is a constant 1.



Find an expression to describe the shadowed region in the figure.

- (a) $|z - z_1| - |z - z_2| > 1$; (b) $\left| |z - z_1| - |z - z_2| \right| > 1$;
 (c) $|z - z_1| - |z - z_2| \leq 1$; (d) $\left| |z - z_1| - |z - z_2| \right| \leq 1$.

8. Let $z = 1 + i$ and $w = 1 + \sqrt{3}i$. Compute $(zw)^3 = \underline{\hspace{2cm}}$.

- (a) $16\sqrt{2}e^{i(\frac{3\pi}{4})}$; (b) $16\sqrt{2}(1 + i)$; (c) $16\sqrt{2}e^{i(-\frac{\pi}{4})}$; (d) $16 + 16i$.

9. For an equation $z^3 = -8$, which of the following is NOT one of its roots?

- (a) $2e^{-i\frac{\pi}{3}}$; (b) -2 ; (c) $1 + \sqrt{3}i$; (d) $-1 + \sqrt{3}i$.

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- 10.** Taking the following photo as an original source of signals, we use an optical Fourier system to modify the low- and high-frequency signals.



If we let only the high-frequency signal pass through (**high-pass**), by filtering out the low-frequency signal, which of the following will be the obtained image photo?

(a)



(b)



(c)



(d) None of the above.

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SECTION B — GAP-FILLING QUESTIONS

Write your answers on the **Examination Book** provided.

This section is worth a total of **20** marks, with each question worth **4** marks.

11. Compute the following double integral

$$I = \iint_D \sqrt{x^2 + y^2} dx dy = \underline{\hspace{2cm}},$$

where D is the region satisfying $x^2 + y^2 \leq 1$ and $y \geq 0$.

12. The so-called *Gamma function* is defined as

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt,$$

which extends the factorial function from the non-negative integers to other real values. In the theory of differential equations we have particular interest in the number $\Gamma\left(\frac{1}{2}\right)$.

Try to evaluate

$$\Gamma\left(\frac{1}{2}\right) = \int_0^\infty t^{\frac{1}{2}-1} e^{-t} dt = \int_0^\infty \frac{e^{-t}}{\sqrt{t}} dt = \underline{\hspace{2cm}}.$$

13. Find all the solutions of the following equation

$$e^z = 1 + i, \quad z \in \mathbb{C}.$$

The solutions are $\underline{\hspace{2cm}}$.

14. Consider a polynomial equation

$$z^4 - 8z^3 + 42z^2 - 80z + 125 = 0, \quad z \in \mathbb{C}.$$

If $z_1 = 1 - 2i$ is one of the roots of this equation, try to find the other roots: $\underline{\hspace{2cm}}$.

15. Consider a function $f(x) = e^{ix}$ with $x \in \mathbb{R}$. If we expand it into a Fourier series,

$$f(x) = e^{ix} = A_0 + \sum_{n=1}^{\infty} A_n \cos(nx) + \sum_{n=1}^{\infty} B_n \sin(nx),$$

then the coefficients A_0, A_n, B_n are determined to be $\underline{\hspace{2cm}}$, where $n \in \mathbb{Z}, n \geq 1$.

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SECTION C — EXTENDED ANSWER QUESTIONS

Write your answers on the **Examination Book** provided.

This section is worth a total of **50** marks. The marks of each question are as shown.

16. (6 marks) Compute the integral

$$I = \int_0^{\infty} \frac{e^{-ax} - e^{-bx}}{x} dx, \quad 0 < a < b.$$

*Hint: $\frac{e^{-ax} - e^{-bx}}{x} = \int_a^b e^{-xy} dy$.

17. (8 marks) Compute the integral

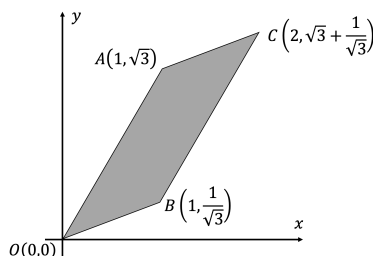
$$I = \int_0^{\frac{\pi}{2}} (\cos^4 \theta + \sin^4 \theta) d\theta,$$

by making use of the Euler formula

$$e^{i\theta} = \cos \theta + i \sin \theta.$$

18. (9 marks) Consider a parallelogram in two dimensions as shown, with the four vertices given by

$$A(1, \sqrt{3}), \quad O(0, 0), \quad B(1, \frac{1}{\sqrt{3}}), \quad C(2, \sqrt{3} + \frac{1}{\sqrt{3}}).$$



Use the polar coordinate system (ρ, θ) to compute the area of this parallelogram, $I = \text{Area}_{\square A O B C}$.

BDIC1043J Advanced Mathematics (Module 4; Finance)**19. (11 marks)** Compute the integral

$$I = \int_0^1 dx \int_0^x dy \int_0^{\sqrt{x^2+y^2}} dz \frac{(x^2 + y^2)^{\frac{3}{2}}}{x^2 + y^2 + z^2}.$$

20. (16 marks) (*From Fourier series to Fourier transform*)

The Fourier series for a periodic function is given by

$$f(x) = A_0 + \sum_{n=1}^{\infty} A_n \cos(nx) + \sum_{n=1}^{\infty} B_n \sin(nx), \quad x = \omega t, \quad \omega = \frac{2\pi}{T}, \quad (1)$$

where

$$\begin{aligned} A_0 &= \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx, & A_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx) dx, \\ B_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx) dx. \end{aligned} \quad (2)$$

The Fourier transform for an aperiodic function is given by

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega) e^{i\omega t} d\omega, \quad (3)$$

with

$$G(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt. \quad (4)$$

Give the **complete derivation procedure** which generalizes the periodic case of Fourier series, eqs.(1)–(2), to the aperiodic case of Fourier transform, eqs.(3)–(4).

Glossary

Aperiodic function	非周期函数
Arbitrary	任意的
Branch	分支
Coefficient	系数
Complete derivation procedure	完整推导过程
Complex plane	复平面
Cone	圆锥
Dashed curve	虚线
Double integral	双重积分
Enclose	包围
Euler formula	欧拉公式
Factorial function	阶乘函数
Finite / Infinite	有限 / 无限
Fourier series	傅立叶级数
Fourier transform	傅立叶变换
Gamma function	伽玛函数
High-frequency	高频
Hyperbola	双曲线
Integrand	被积函数
Limitation behavior	极限行为
Low-frequency	低频
Optical	光学的
Original source of signal	信号源
Parallelogram	平行四边形
Partition	分割
Periodic function	周期函数
Polynomial equation	多项式方程
Region	区域
Root	(方程的) 根
Shadowed region	阴影区域
Singular point	奇点
Spherical coordinate	球坐标
Triple integral	三重积分
Unbounded	无界
Vertex (<i>pl.</i> Vertices)	顶点